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Lie algebras which have an ascending series with simple factors

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Introduction

In this paper we investigate Lie algebras which have an ascending series whose factors are simple. Here simple Lie algebras are non-abelian simple. In [3] Levich has shown that if H is an ascendant subalgebra of a simple Lie algebra L, then $H=0$ or $H=L$. In particular H is a perfect characteristic ideal of L. In § 1 we shall show that, in a Lie algebra which has an ascending series whose factors are simple, every ascendant subalgebra is a perfect characteristic ideal. In § 2 we consider a special case and its application. In [6] it has been shown that, in the Lie algebra L of all endomorphisms of an infinite-dimensional vector space, every subideal is an ideal of L. We shall show in § 2 that every ascendant subalgebra of L is an ideal. In § 3 we shall show that $\acute{\mathrm{E}}(\lhd)\mathfrak{F} \leq \mathrm{L}\mathfrak{F}$. Using the results of §§ 1 and 3, we shall show in § 4 that in a Lie algebra which has an ascending series whose factors are finite-dimensional simple, every serial subalgebra is a perfect characteristic ideal. In § 5 we apply our results to prove that, in a semi-simple neoclassical algebra, serial subalgebras and local subideals are perfect characteristic ideals. In [7] it has been shown that every soluble Lie algebra, in which every ascendant subalgebra is an ideal, is either abelian or the split extension of an abelian Lie algebra by the 1-dimensional algebra of scalar multiplications and conversely. We shall finally show in § 6 that in the split extension of an abelian Lie algebra by the 1-dimensional algebra of scalar multiplications every serial subalgebra is an ideal.

Let H be a subalgebra of a Lie algebra L and let Σ be a totally ordered set. A series from H to L of type Σ is a family $\{\Lambda_\sigma, V_\sigma : \sigma \in \Sigma\}$ of subalgebras of L such that

(1) For all σ, $H \leq \Lambda_\sigma$ and $H \leq V_\sigma$,

(2) $L \setminus H = \cup_{\sigma \in \Sigma} (\Lambda_\sigma \setminus V_\sigma)$,

(3) $\Lambda_\tau \leq V_\sigma$ if $\tau < \sigma$,

(4) $V_\sigma \lhd \Lambda_\sigma$.

The quotient algebras $\Lambda_\sigma / V_\sigma$ are the factors of the series. If Σ is well-ordered (resp. reversely well-ordered, finite), then the series is called an ascending series (resp. a descending series, a subideal) and we write $H \operatorname{ser} L$ (resp. $H \operatorname{desc} L$,