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Asymptotic values of meromorphic functions of smooth growth

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1. Introduction

In the following, the standard notation of Nevanlinna theory (e.g., see Hayman [7]) will be used.

Hayman [8] gave a striking example of a meromorphic function $f(z)$ in the whole plane such that $\delta(\infty, f)=1$ but ∞ is not an asymptotic value of $f(z)$. To point out that the singular behaviour of this $f(z)$ is essentially associated with the irregular growth of Nevanlinna characteristic $T(r, f)$, he picked up several sorts of smoothly growing conditions of $T(r, f)$, under which certain deficient values are asymptotic values.

In [8, Corollary 2], Hayman proved that, if a meromorphic function $f(z)$ satisfies the smoothness condition

$$(1) \qquad T(2r, f) \sim T(r, f) \qquad (r \to \infty),$$

then any deficient value of $f(z)$ is an asymptotic value of $f(z)$. Further, extending the result [3, Theorem 4] and answering to the question [2, 2.57], Anderson [1] proved that for $f(z)$ satisfying (1), if w is a deficient value of $f(z)$, we can find a path Γ going to ∞ and satisfying

$$(2) \qquad L(r, \Gamma) = r(1 + o(1)) \qquad (r \to \infty)$$

along which

$$\liminf_{|z| \to \infty} (\log 1/|f(z) - w|)/T(|z|, f) \geqq \delta(w, f) \qquad (w \neq \infty)$$

$$\liminf_{|z| \to \infty} (\log |f(z)|)/T(|z|, f) \geqq \delta(w, f) \qquad (w = \infty)$$

where $L(r, \Gamma)$ is the length of the arc $\Gamma \cap \{z: |z| \leqq r\}$.

The aim of this paper is mainly to extend this Anderson's result to meromorphic functions of positive order ρ ($\rho < 1/2$) satisfying the smoothness condition

$$(3) \qquad \limsup_{r \to \infty} x^{-\rho} T(r, f)^{-1} T(xr, f) \leqq 1$$

for any x ($x > 1$), because meromorphic functions satisfying (1) have order 0 (see Hayman [8, p. 130]). But, we could not get any result corresponding to (2),