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On a certain class of irreducible unitary representations of the infinite dimensional rotation group I

Dedicated to Professor Y. Matsushima for his 60th birthday

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Introduction

The purpose of this paper is to show that the McKean's conjecture in [2] is valid for the set of all equivalence classes of irreducible unitary representations of class one.

§1. Spherical functions

Let $\boldsymbol{H}$ be a separable Hilbert space over $\boldsymbol{R}$ (or $\boldsymbol{C}$). In this paper, we fix, once for all, an orthonomal basis $\{\xi_j; j \in \boldsymbol{N}\}$ of $\boldsymbol{H}$, where $\boldsymbol{N}$ is the set of all positive integers. Let $\boldsymbol{E}$ be the space algebraically spanned by the basis $\{\xi_j; j \in \boldsymbol{N}\}$. We denote by $\boldsymbol{E}_m$ the space spanned by the set $\{\xi_j; j=1,\dots, m\}$. Then we have $\boldsymbol{E}=\bigcup_{m=1}^{\infty} \boldsymbol{E}_m$. Since a countable inductive limit of nuclear spaces is nuclear, $\boldsymbol{E}$ is a nuclear space. Let G be the group of all isometries g of $\boldsymbol{H}$ such that $g\xi_j=\xi_j$ except finitely many j in $\boldsymbol{N}$. We denote by G_m the group of all elements g in G such that $g\xi_j=\xi_j$ $(j=m+1, m+2,\dots)$. Then we have $G=\bigcup_{m=1}^{\infty} G_m$. By the inductive limit topology G is a topological group. For a g in G_m, putting $g\xi_j=\sum_{i=1}^{m} g_{ij}\xi_i$ $(j=1,\dots, m)$, we can identify g with the matrix (g_{ij}) in $O(m)$ (or $U(m)$).

We denote by $\boldsymbol{E}^*$ the dual space of $\boldsymbol{E}$, then we have a triple

$$\boldsymbol{E} \subset \boldsymbol{H} \subset \boldsymbol{E}^*.$$

By the Bochner-Minlos theorem, there exists a probability measure μ on $\boldsymbol{E}^*$ such that for any ξ in $\boldsymbol{E}$ we have

$$e^{-\|\xi\|^2/2} = \int_{\boldsymbol{E}^*} e^{i\langle x,\xi\rangle}\, d\mu(x). \tag{1.1}$$

We use the same notation for the dual action of g on $\boldsymbol{E}^*$. Clearly μ is G-invariant. For any g in G and f in $L^2(\boldsymbol{E}^*, \mu)$ we define

$$(\pi_*(g)f)(x) = f(g^{-1}x) \quad \text{for a.e. } x \text{ in } \boldsymbol{E}^*.$$

Then it is easy to see that π_* is a unitary representation of G on $L^2(\boldsymbol{E}^*, \mu)$. For