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A classification of certain symmetric Lie algebras

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Introduction

Harish-Chandra's profound investigations tell us many important facts concerning semisimple Lie groups. The following is one of them; a connected real semisimple Lie group G has discrete series if and only if G has a compact Cartan subgroup. If a connected noncompact real simple Lie group has a compact Cartan subgroup, its Lie algebra is of inner type. One can find the classification of real simple Lie algebras of inner type in Murakami [6]. For affine symmetric spaces, in [3] Flensted-Jensen has proved a similar result to Harish-Chandra's. That is to say; if an affine symmetric space G/H of a connected noncompact real semisimple Lie group G has a compact Cartan subspace, then the regular representation of G on $L^2(G/H)$ contains closed invariant subspaces. Recently, in [5] Matsumoto has given a sufficient condition for that;

(*) Some representations belonging to the discrete series of a connected semisimple Lie group G appear as closed invariant subspaces in the regular representation of G on $L^2(G/H)$. Here $L^2(G/H)$ is the space of all square integrable functions on an affine symmetric space G/H with respect to the invariant measure.

In this article, we shall sort symmetric real simple Lie algebras which satisfy the condition in [5]. For this, we shall describe the condition in terms of the root theory. Let $(\mathfrak{g}, \sigma)$ be a pair consisting of a real simple Lie algebra and its involutive automorphism (the so-called symmetric Lie algebra). In our case $\mathfrak{g}$ may be determined by a Dynkin diagram and a simple root, and σ will be characterized by a Satake diagram. A classification of all symmetric real simple Lie algebras has been shown by Berger, so that we will search the list in [2]. Also, Flensted-Jensen has obtained a sufficient condition for (*) ([3], Theorem 7.14). In view of the classification, his condition seems different from Matsumoto's.

Throughout the paper, we assume that Lie algebras are defined over the field of real numbers, and we denote the dual space of a real or complex vector space V by V^*. In addition, we denote by V_c the complexification of a real vector space V.

1. Preliminaries

This section is devoted to recalling some basic terminology. Let $\mathfrak{g}$ be a