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Boundary limits of Green potentials of order α

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1. Introduction

In the half space $D=\{x=(x_1,\dots, x_n);\ x_n>0\}$, $n\geqq 2$, the Green potential of order α, $0<\alpha<n$, of a non-negative measurable function f on D is defined by

$$G_\alpha^f(x) = \int_D G_\alpha(x, y)f(y)dy,$$

where $G_\alpha(x, y)=|x-y|^{\alpha-n}-|\bar{x}-y|^{\alpha-n}$, $\bar{x}=(x_1,\dots, x_{n-1}, -x_n)$ for $x=(x_1,\dots, x_{n-1}, x_n)$. Our aim in this note is to study the existence of boundary limits of G_α^f. One of our results is as follows:

Let $p>1$, $\gamma<2p-1$ and f satisfy $G_\alpha^f\not\equiv\infty$ and

$$\int_G f(y)^p y_n^\gamma dy < \infty \qquad \text{for any bounded open set} \quad G\subset D.$$

Then there exists a set $E\subset\partial D$ with $H_{n-\alpha p+\gamma}(E)=0$ such that to each $\xi\in\partial D-E$, there corresponds a set $E_\xi\subset S_+=\{x\in D;\ |x|=1\}$ with the properties:

a) $B_{\alpha,p}(E_\xi)=0$; b) $\lim_{r\downarrow 0} G_\alpha^f(\xi+r\zeta)=0$ *for every* $\zeta\in S_+-E_\xi$,

where H_ℓ denotes the ℓ-dimensional Hausdorff measure and $B_{\alpha,p}$ denotes the Bessel capacity of index (α, p) (see [3]).

In case $\alpha=2$, according to Wu [8; Theorem 1], the exceptional set E_ξ has Hausdorff dimension at most $n-2p$; this is a consequence of our result in view of Fuglede [2].

Moreover, non-tangential limits, fine limits, mean continuous limits and perpendicular limits will be considered.

2. Preliminaries

Let us begin with the following lemma, which can be proved by elementary calculation.

LEMMA 1. *There exist positive constants c_1 and c_2 such that*

$$c_1\frac{x_n y_n}{|x-y|^{n-\alpha}|\bar{x}-y|^2} \leqq G_\alpha(x, y) \leqq c_2\frac{x_n y_n}{|x-y|^{n-\alpha}|\bar{x}-y|^2}$$

for $x=(x_1,\dots, x_n)$ and $y=(y_1,\dots, y_n)$ in D.