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Bernstein's theorem and translation invariant operators

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We propose to give non-homogeneous versions of some results on Fourier transforms and translation invariant operators of homogeneous Besov and Hardy spaces. Our first aim is to derive an analogue of Herz's version of Bernstein's theorem ([8]) for the non-homogeneous Besov spaces of Taibleson ([15]). Two proofs of this theorem will be presented. The first proof is quite elementary; the main tool is an inequality due to Flett ([5]). Our second proof is based on a relation linking non-homogeneous and homogeneous spaces, which allows us to pass from non-homogeneous to homogeneous spaces and then to use the theorem of Herz. As it turns out, the spaces describing integrability of Fourier transforms of distributions in non-homogeneous Besov spaces arise naturally as intermediate spaces between weighted L^p spaces ([6]), and they also generalize some algebras of Beurling ([2]).

Our second group of results concerns translation invariant operators. It is a known fact that Besov spaces can be used to measure smoothness of translation invariant operators on Lebesgue or Besov spaces ([15], [14]). The results of Johnson ([10], [11]) give necessary and/or sufficient conditions, in terms of homogeneous Besov spaces, for operators on Hardy spaces to be bounded and translation invariant. We generalize these results to the local Hardy spaces defined in [7], and improve or supplement some results of Taibleson and Stein-Zygmund ([15], [14]).

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NOTATION. Our notation is standard. We use R^n to denote the n-dimensional euclidean space and R^{n+1}_+ to denote the cartesian product $R^n \times]0, \infty[$. An element of R^{n+1}_+ is denoted by (x, t), where $x \in R^n$ and $0<t<\infty$. The Fourier transform is defined by

$$\mathscr{F} f(x) = \hat{f}(x) = \int e^{-2\pi i x \cdot y} f(y) dy ,$$

where $x \cdot y = x_1 y_1 + \cdots + x_n y_n$, and the integral is extended over all of R^n unless otherwise indicated. If u is an infinitely differentiable function on R^{n+1}_+, then $D^k_{n+1} u$ stands for $(\partial/\partial t)^k u$. For measurable functions u on R^{n+1}_+, we write