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Minimal conditions for weak subideals of Lie algebras

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1.

Minimal conditions for subideals of Lie algebras were investigated in [1] and [3]. Among other things the following result was shown:

$$\bigcap_{n=1}^{\infty} \text{Min-}\lhd^{n} \mathfrak{X} = \text{Min-si}\, \mathfrak{X}$$

for any I-closed class $\mathfrak{X}$ of Lie algebras. On the other hand, we introduced the notion of weak subideals in [4], which generalizes the notion of subideals. Thus in this paper we shall establish the similar result concerning the minimal conditions for weak subideals of Lie algebras.

2.

Throughout the paper we employ the notations and terminology in [1] and [4], and all Lie algebras are over a field of arbitrary characteristic.

We denote by Min-wsi (resp. Min-$\leq^{n}$) the class of Lie algebras satisfying the minimal condition for weak subideals (resp. n-step weak subideals). For a class $\mathfrak{X}$ of Lie algebras we denote by Min-wsi $\mathfrak{X}$ (resp. Min-$\leq^{n} \mathfrak{X}$) the class of Lie algebras which satisfy the minimal condition for weak subideals (resp. n-step weak subideals) belonging to $\mathfrak{X}$. Similarly, we define Min-wasc $\mathfrak{X}$ and Min-$\leq^{\alpha} \mathfrak{X}$ where α is an ordinal.

We call a class $\mathfrak{X}$ of Lie algebras WSI-closed if H wsi $L \in \mathfrak{X}$ implies $H \in \mathfrak{X}$. Hence $\mathfrak{X}$ is WSI-closed if it is S-closed.

Now we shall state the following three lemmas, which can be shown easily.

LEMMA 1 ([4]). *Let L be a Lie algebra and let m, n be any integers ≥ 0. Then*:

(a) *If $H \leq^{m} K \leq^{n} L$, then $H \leq^{m+n} L$.*

(b) *If $H \leq^{m} L$ and $K \leq L$, then $H \cap K \leq^{m} K$.*

(c) *Let f be a homomorphism from L onto a Lie algebra $\bar{L}$. If $H \leq^{m} L$, then $f(H) \leq^{m} \bar{L}$.*

LEMMA 2. Min-wsi *is* E-*closed.*

LEMMA 3 ([2]). *If H* wsi *L, then $H^{\omega} = \bigcap_{n=1}^{\infty} H^{n} \lhd L$.*