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Balanced fractional $r^m \times s^n$ factorial designs and their analysis

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1. Introduction and summary

The theory of a fractional factorial design was originated by Fisher [18], who treated the development of confounding systems for factorial designs (cf. [17, 40]), and further Finney [16] gave the first definitive approach. This theory takes aim at the search of "good" fractional factorial designs (cf. [14, 19]). There are many criteria of *goodness*, some of which are:

A. *Save the number of assemblies* (*treatment combinations*).
B. *Estimate the unknown effects independently.*
C. *Minimize the value of some function $f(T)$ on a class of designs T having the same size* (*the number of assemblies*) *N, where $f(T)$ evaluates a sort of the loss of the information.*

As $f(T)$, the following types are used commonly:

$$\det(V_T),\ \mathrm{tr}(V_T) \text{ and the } \textit{maximum characteristic root of } V_T,$$

where $\sigma^2 V_T$ is the variance-covariance matrix of the estimates of the effects based on a design T. These optimality criteria are called the determinant, trace and maximum root criteria, respectively. They aim to minimize the volume of a confidence region for the effects of interest, the average variance, and the largest variance of the estimates of all normalized linear combinations of the effects, respectively (cf. [33]).

The complete design satisfies the *criteria B* and *C*, but it needs a large number of assemblies, which imply that the complete design is unreasonable in the sense of the *criterion A*. An *orthogonal design*, defined by Rao [27] in s^m factorials in which each of m factors has s levels, satisfies the *criteria* B and C (cf. [1, 4, 6, 15, 20, 26]). This design can reduce the number of assemblies in comparison with the complete design. However, an orthogonal design exists only for special values of the size, and the use of such a design may be, in general, uneconomic in the sense that it involves more than the desirable size. For an example of 2^7 factorials of resolution V (the term *resolution* was defined by Box and Hunter [2]), an orthogonal design needs $2^6=64$ or $2^7=128$ (the complete design) assemblies since there exists no orthogonal design of size $2^5=32$ and of