

Bipartite decomposition of complete multipartite graphs

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1. Introduction

Graph theory is a subject of combinatorics in mathematics and it is one of the most flourishing branches of modern algebra with wide applications to various fields. The problem of decomposing a graph into a union of subgraphs each isomorphic to a given graph is an important subject of graph theory. There are many types of decomposition problems, such as, clique decomposition [7, 15], claw decomposition [18, 19, 20, 22, 24], path decomposition [9, 13, 14], cycle decomposition [4, 6, 16], bipartite decomposition [10, 11] and so on. Some of them are used, for example, for combinatorial file organization schemes in filing theory and some are used for construction schemes of designs of experiments in statistics.

We are concerned with a bipartite decomposition, which includes a claw decomposition as a special type. It will be used for a design of combinatorial file organization scheme.

Some results [5, 10, 11, 17, 24] are known about the decompositions of a complete graph K_m with m points. The problem of claw decomposition of a complete graph K_m has been raised and solved completely by Yamamoto, Ikeda, Shige-eda, Ushio and Hamada [24]. The claw decomposition of a complete graph provides us a balanced file organization scheme of order two for binary-valued records. It is optimal in such a sense that it has the least redundancy among all possible balanced binary-valued file organization schemes of order two having the same parameters, provided the distribution of records has the property of invariance with respect to the permutation of attributes. Such a scheme is called HUBFS₂ [25]. Huang and Rosa [10] and Huang [11] have investigated a bipartite decomposition of a complete graph K_m by introducing the concept of the balance of points.

As for the decomposition of a complete multipartite graph, many authors [18, 19, 20, 21, 22, 24] have studied. The complete solution of the problem of claw decomposition of a complete bipartite graph has been given by Yamamoto et al. [24]. Ushio, Tazawa and Yamamoto [20] have given a theorem which states a necessary and sufficient condition for a complete m -partite graph $K_m(n, \dots, n)$ with m sets of n points each to have a claw decomposition. Moreover, Tazawa, Ushio and Yamamoto [18] have given a necessary and sufficient condition for a