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Compact transformation groups on Z_2-cohomology spheres with orbit of codimension 1

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§ 1. Introduction

Let M be a connected closed smooth manifold and G be a compact connected Lie group which acts smoothly on M, and consider the following assumption:

(AI) *There is an orbit* $G\cdot x$ *of* $x\in M$ *such that* $\dim G\cdot x=\dim M-1$.

Then the following is well-known (cf., e.g., [4; IV, Th. 3.12, Th. 8.2]):

(1.1) *For a G-action on M with* (AI), *where M is simply connected, there is a triple* (K, K_1, K_2) *of subgroups of G with* $K\subset K_1\cap K_2$ *such that K is a principal isotropy subgroup with* $\dim G/K=n-1$ $(n=\dim M)$, K_1 *and* K_2 *are non-principal ones with* $k_s=n-\dim G/K_s\geqq 2$ $(s=1, 2)$, *and the G-manifold M can be decomposed into the union of two mapping cylinders of the projections* $G/K\to G/K_s$ $(s=1, 2)$. (See (3.2–6).)

Based on (1.1), such actions are studied by several authors. For example, H. C. Wang [15] investigated such actions on the spheres S^n with even $n\neq 4$ or odd $n\geqq 33$, and W. C. Hsiang and W. Y. Hsiang [7] have given some examples which are not listed in [15].

The purpose of this paper is to classify such actions (G, M) with (AI) for the case that M is a Z_2*-cohomology sphere*, i.e.,

(AII) *M is simply connected and* $H^*(M; Z_2)\cong H^*(S^n; Z_2)$.

Typical examples of such (G, M) are seen among the linear actions (G, S^n, ψ) on S^n via representations $\psi: G\to SO(n+1)$. Moreover, we have the following example due to W. C. Hsiang and W. Y. Hsiang:

EXAMPLE 1.2 ([7; Example 5.3], cf. [4; Ch. I, § 7 and Ch. V, § 9]). For any odd integer $r\geqq 1$, consider the $(2m-1)$-manifold

$$W^{2m-1}(r) = \{(z_0, z)\in C\times C^m;\ |z_0|^2+|z|^2 = 2,\ z_0^r+z\cdot {}^t z = 0\}.$$

Then, this is a Z_2-cohomology sphere. Further, for any subgroup G of $SO(m)\times S^1$, the G-action on $W^{2m-1}(r)$ is defined by