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On the trace mappings in the space $B_{1,\mu}(R^N)$

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Let μ be a temperate weight function on $\Xi^N=(R^N)'$, that is, a positive valued continuous function on Ξ^N such that

$$\mu(\xi+\eta) \leqq C(1+|\xi|^k)\mu(\eta), \quad \xi, \eta \in \Xi^N$$

with positive constants k and C[4, p. 7]. By $B_{p,\mu}(R^N)$, $1 \leqq p \leqq \infty$, we denote the set of all temperate distributions $u \in \mathscr{S}'(R^N)$ such that its Fourier transform $\hat{u}$ is a locally summable function and

$$\|u\|_{p,\mu}^p = (2\pi)^{-N}\int_{\Xi^N} |\hat{u}(\xi)|^p \mu^p(\xi) d\xi < \infty,$$

and when $p=\infty$ we shall interpret $\|u\|_{\infty,\mu}$ as ess. sup $|\hat{u}(\xi)\mu(\xi)|$ [1, p. 36].

In our previous papers [2, 3] we have investigated the trace mappings in the space $B_{p,\mu}(R^N)$ with $1<p<\infty$. The purpose of this paper is to develop the analogues of the theorems in [3] for the space $B_{1,\mu}(R^N)$.

Let $N=n+m$. We shall use the notations: $x=(x', t) \in R^N$, $x'=(x_1',\dots, x_n')$, $t=(t_1,\dots, t_m)$ and $\xi=(\xi', \tau) \in \Xi^N$, $\xi'=(\xi_1',\dots, \xi_n')$, $\tau=(\tau_1,\dots, \tau_m)$. For a polynomial $P(\xi)=\Sigma a_\alpha \xi^\alpha$ in ξ, we put $\bar{P}(\xi)=\Sigma \bar{a}_\alpha \xi^\alpha$ and $P(D)=\Sigma a_\alpha D^\alpha$ with $D=(D_1,\dots, D_N)$, $D_j=-i\partial/\partial_j$. $P^{(\alpha)}$ means $i^{|\alpha|}D^\alpha P$. Let μ_1 and μ_2 be temperate weight functions on Ξ^N. Then $\mu_1+\mu_2$, $\mu_1\mu_2$ and $1/\mu_1$ are temperate weight functions on Ξ^N.

If μ is a positive valued function on Ξ^N satisfying the inequality

$$\mu(\xi+\eta) \leqq (1+C|\xi|)^k \mu(\eta), \quad \xi, \eta \in \Xi^N$$

with positive constants k and C, then we have

$$(1+C|\xi|)^{-k} \leqq \mu(\xi+\eta)/\mu(\eta) \leqq (1+C|\xi|)^k,$$

which implies the continuity of μ[1, p. 34]. Putting $\nu(\xi')=\sup_\tau \mu(\xi', \tau)$, we have $\nu(\xi'+\eta') \leqq (1+C|\xi'|)^k \nu(\eta')$ for any $\xi', \eta' \in \Xi^n$.

Let μ be the function defined on Ξ by $\mu(\xi)=1$ for $\xi \leqq 0$, $\mu(\xi)=1+(2\xi-\xi^2)^{1/2}$ for $0<\xi<1$ and $\mu(\xi)=2$ for $\xi \geqq 1$. Then μ is a temperate weight function but it does not satisfy the inequality $\mu(\xi+\eta) \leqq (1+C|\xi|)^k \mu(\eta)$ with positive constants k and C. If $\mu(\xi)=1+\arg(\xi'+ie^\tau)$ on Ξ^2, then μ is a temperate weight function but $\nu(\xi')=\sup_\tau \mu(\xi', \tau)$ is not continuous.

According to L. Hörmander [1, p. 36] we shall first prove