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On the decomposition of homogeneous systems with nondegenerate Killing-Ricci tensor

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§ 0. Introduction

The notion of homogeneous systems has been introduced in [4]. For the investigation of various properties of homogeneous Lie loops and their tangent Lie triple algebras introduced in [1], it seems to be more convenient to treat them as homogeneous systems. In the present paper, the decomposition theorem of analytic homogeneous systems is shown in such a case that the canonical connection satisfies a tensor equation (2.4) and that the Killing-Ricci form of the tangent Lie triple algebra is nondegenerate (Theorem and Corollary 1 in § 2). For the symmetric homogeneous systems we get the de Rham-Wolf decomposition of the pseudo Riemannian structure defined by the Ricci tensor (Corollary 2 in § 2).

§ 1. Preliminaries

An *analytic homogeneous system* (G, η) is a connected analytic manifold G together with an analytic ternary operation $\eta: G \times G \times G \to G$ satisfying (1) $\eta(x, y, x) = \eta(x, x, y) = y$, (2) $\eta(x, y, \eta(y, x, z)) = z$ and (3) $\eta(x, y, \eta(u, v, w)) = \eta(\eta(x, y, u), \eta(x, y, v), \eta(x, y, w))$. Let (G, η) be an analytic homogeneous system. For $x, y \in G$, the analytic diffeomorphism $\eta(x, y)$ of G defined by $\eta(x, y)z = \eta(x, y, z)$ is called the *displacement* of (G, η) from x to y. Let G be an analytic *homogeneous Lie loop*, i.e., a loop with an analytic multiplication xy satisfying the conditions (1) the left translations and right translations are all analytic diffeomorphisms of G, (2) there exists a two sided identity element e, (3) for each x there exists a two sided inverse element x^{-1} of x such that $L_x^{-1} = L_{x^{-1}}$, where L_x denotes the left translation by x, and (4) the *left inner mapping* $L_{x,y} = L_{xy}^{-1} L_x L_y$ is an automorphism of the loop G for any $x, y \in G$. For such a homogeneous Lie loop G an analytic homogeneous system η can be defined on G by

$$(1.1) \qquad \eta(x, y, z) = x((x^{-1}y)(x^{-1}z)) \qquad \text{for} \quad x, y, z \in G.$$

Any analytic homogeneous system (G, η) with a base point e is a local homogeneous Lie loop under the multiplication

$$(1.2) \qquad xy = \eta(e, x, y)$$