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# Novikov's Ext² at the prime 2

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## §1. Introduction

Let $BP$ denote the Brown-Peterson spectrum at a prime $p$, whose coefficient ring $BP_*=\pi_*(BP)$ is the polynomial ring

$$BP_* = Z_{(p)}[v_1, v_2, \cdots] \qquad (\deg v_i = 2(p^i-1))$$

with Hazewinkel's generators $v_i$ ([2]). Then, we have the Hopf algebroid $(BP_*, BP_*BP)$, where $BP_*BP=BP_*[t_1, t_2, \cdots]$ $(\deg t_i=2(p^i-1))$ ([1; Part II], [5], [8], [6; §1]). For the spectrum $BP$, we have Novikov's analogue of the Adams spectral sequence converging to the stable homotopy ring $\pi_*(S)$ of the sphere spectrum $S$. Its $E_2$-term $E_2^{*,*}$ is the cohomology $\mathrm{Ext}^{*,*}_{BP_*BP}(BP_*, BP_*)$ (denoted simply by $\mathrm{Ext}^{*,*}BP_*$) of the Hopf algebroid $(BP_*, BP_*BP)$, (cf. [1; Part III], [7]). The $E_2$-term $E_2^{1,*}$ is determined by S. P. Novikov [7] for any prime $p$, and $E_2^{2,*}$ by H. R. Miller, D. C. Ravenel and W. S. Wilson [6] for any odd prime $p$.

In this paper, we shall determine $E_2^{2,*}$ for the prime 2, and study the non-triviality of some elements in $\pi_*(S)$. We notice that the results for $E_2^{2,*}$ is also obtained by S. A. Mitchell, independently. From now on, we assume that the prime $p$ is 2.

To state our results, we recall the elements $y_i \in v_1^{-1}BP_*$ and $x_i \in v_2^{-1}BP_*$, which are denoted by $x_{1,i}$ and $x_{2,i}$ in [6; (5.11)] respectively, given by

(1.1) $$y_0 = v_1,\ y_1 = v_1^2 - 4v_1^{-1}v_2,\ y_i = y_{i-1}^2 \quad (i \geqq 2),$$
$$x_0 = v_2,\ x_1 = v_2^2 - v_1^2v_2^{-1}v_3,\ x_2 = x_1^2 - v_1^3v_2^3 - v_1^5v_3,\ x_i = x_{i-1}^2 \quad (i \geqq 3).$$

By using these elements and the universal Greek letter map $\eta$ (for the definition, cf. [6; (3.6)]), we can define the elements

(1.2) $$\alpha_m = \eta(v_1^m/2) \text{ for odd } m \geqq 1, \qquad \alpha_{2/2} = \eta(y_1/4), \text{ and}$$
$$\alpha_{2^im/i+2} = \eta(y_i^m/2^{i+2}) \text{ for } i \geqq 1, \text{ odd } m \geqq 1 \text{ with } 2^im \geqq 4,$$

which generate $\mathrm{Ext}^1BP_*$ (cf. [6; Cor. 4.23]). Further, we can define the elements

(1.3.1) $$\beta_{2^ns/j,i+1} = \eta(x_n^s/2^{i+1}v_1^j) \quad \text{in} \quad \mathrm{Ext}^2BP_*$$

for $n \geqq 0$, odd $s \geqq 1$, $j \geqq 1$, $i \geqq 0$ with