Convergence of difference approximations for
quasi-linear hyperbolic systems

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1. Introduction

From numerical aspects, difference schemes for solving initial value problems have been extensively investigated for a long time. In linear initial value problems, Lax obtained the remarkable result which states that the stability of a consistent difference scheme is a necessary and sufficient condition for the convergence if the problem is well posed [22]. This has become well known as Lax's Equivalence Theorem. The stability used there means conceptually the boundedness of numerical solutions given by the difference scheme. For numerical studies of engineering and physical problems, many authors have constructed several difference schemes. In particular, Peetre and Thomée [21] investigated these in the Sobolev spaces and gave several estimates for the rate of convergence, assuming that they are stable. For linear hyperbolic systems the theory of the Sobolev spaces of L_2-type has played an important role in establishing the existence and uniqueness of solution. Along such lines the L_2-stability of difference schemes has also been studied. Such a stability can be obtained in terms of the amplification matrix defined by the Fourier transform of the difference operator. Lax [13], Lax and Wendroff [14, 15], Kreiss [11], Yamaguti and Nogi [31], Lax and Nirenberg [16], Vaillancourt [28, 29, 30], Koshiba [10] and so on [20, 25, 26, 27] are relevant here.

Though a considerable portion of the progress in difference schemes for hyperbolic systems is confined to the linear theory, we must mention some results related to nonlinear theory of initial value problems. Strang [24] contributed to the establishment of convergence of difference approximations to smooth solutions in nonlinear problems. This will be briefly stated below. Modifying Lax's Equivalence Theorem, Kreth [12], von Dein [3] and Ansorge [1] discussed the convergence of difference approximations in an abstract setting and so, to our knowledge, their results seem less applicable.

Concerning weak solutions of quasi-linear hyperbolic equations of conservation laws, Le Roux [17] studied problems of the convergence for difference approximations.

In nonlinear initial value problems, Lax's Equivalence Theorem is not valid in general, so the convergence must be proved without the help of the stability