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Some results on the normalization and normal flatness

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Introduction

In this paper, we shall give a sufficient condition that properties for a reduced noetherian scheme X to be Cohen-Macaulay or Gorenstein can be ascended to or can be descended from the same properties on the normalization $\bar{X}$ of X. It is well-known that the condition of flatness plays an important role in the study of many properties on an extension of a noetherian rings (e.g. [21]). But the normalization of a reduced noetherian ring is an integral extension which is far from a flat one. Therefore it seems to the author that we need a "flatness" condition on X, in some sense, in order to give the above sufficient condition. Fortunately, in his famous paper [11], H. Hironaka defined the notion of normal flatness in 1964 (see Def. 2 in this paper). From that time, many mathematicians have studied properties on normal flatness and have obtained many results on it (e.g. [9], [10]). Let Y be the closed subscheme of X defined by the conductor of X in $\bar{X}$. By the definition of normal flatness, if X is normally flat along Y, that is, if the normal cone N of X along Y is flat over Y, then $X' \times_X Y$ is flat over Y where X' is the blowing up of X along Y. On the other hand, there is a canonical morphism from X' to $\bar{X}$ (see Prop. 3 in this paper) and P. H. Wilson showed, in the case where X is a hypersurface, that a necessary and sufficient condition for this canonical morphism to be an isomorphism can be spoken by a "flatness" condition (cf. Theorem 2.7 in his paper [22]). The author believes that, under the condition that X is normally flat along Y, the fibres of N along Y and hence the fibres of X' along Y are well parametrized. In this point of view, we shall study the structure of N and show that *if X is normally flat along Y and Y is of pure codimension* 1 *in X, then*

(i) *X' is naturally isomorphic to $\bar{X}$.*

(ii) *$\bar{X}$ is a Cohen-Macaulay scheme if and only if so is X.*

(iii) *$\bar{X}$ is a Gorenstein scheme if so is X.*

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