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Symplectic Pontrjagin numbers and homotopy groups of $MSp(n)$

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Introduction

In [10] and [11], E. Rees and E. Thomas have studied the divisibility of some Chern numbers of the complex cobordism classes and the homotopy groups of $MU(n)$. The purpose of this paper is to study the symplectic cobordism theory by using their methods.

Let $MSp(n)$ be the Thom space of the universal symplectic vector bundle over the classifying space $BSp(n)$, and $MSp=\{MSp(n), \varepsilon_n\}$ be the Thom spectrum of the symplectic cobordism theory, where $\varepsilon_n: \Sigma^4 MSp(n) \to MSp(n+1)$ is the structure map. Let $b_n: MSp(n) \to \Omega^{4N} MSp(n+N)$ be the adjoint map of the composition $\varepsilon_{n,N}: \Sigma^{4N} MSp(n) \to MSp(n+N)$ of $\Sigma^i \varepsilon_{n+i}$, where $N \geqq n > 0$. Converting b_n into a fibering with fiber F_n, we consider the fibering

$$(1) \qquad F_n \longrightarrow MSp(n) \xrightarrow{b_n} \Omega^{4N} MSp(n+N).$$

Then F_n is $(8n-2)$-connected, and we can determine the cohomology groups of F_n in dimensions less than $12n-2$ (see Proposition 2.15).

Let $P_i \in H^{4i}(BSp)$ be the i-th symplectic Pontrjagin class. For a symplectic cobordism class $u \in \pi_{4k}(MSp)$ and a class $P_{i_1} \cdots P_{i_j} \in H^{4k}(BSp)$ with $\sum_{t=1}^{j} i_t = k$, $P_{i_1} \cdots P_{i_j}[u]$ denotes the Pontrjagin number of u for a class $P_{i_1} \cdots P_{i_j}$.

Our first purpose is to obtain the divisibility of some Pontrjagin numbers of the symplectic cobordism classes by making use of the cohomology groups of F_n. As a concrete result, we have the following theorem (see Theorem 3.8):

THEOREM I. *Let* $n \geqq 1$. *Then*
(i) $P_n[u] \equiv 0 \mod 8$ *for any* $u \in \pi_{4n}(MSp)$.
(ii) $P_1 P_n[u] - ((n+4)/2) P_{n+1}[u] \equiv 0 \mod 24$ *for any* $u \in \pi_{4n+4}(MSp)$.

The divisibility of Pontrjagin numbers of some symplectic cobordism classes has been studied in [14], [13], [3], [6] to investigate the structure of $\pi_*(MSp)$. For the divisibility (i) of the above theorem, E. E. Floyd [3] has proved it with some restriction by using the alternative method, and some application of the method of Floyd is considered in [4].

The second purpose of this paper is to study the homotopy groups