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Special concircular vector fields in Riemannian manifolds

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Introduction

The purpose of the present paper is to study Riemannian manifolds admitting some linearly independent special concircular vector fields and determine geometrical structures of such manifolds. Some results in this paper contain generalizations of results due to Y. Tashiro (see Proposition 7.3 in [4] and Corollaries 2 and 3 in this paper).

We shall define an almost everywhere warped product and give a few examples in § 1. We also state some properties of this kind of product. In § 2, we shall determine structures of n-dimensional Riemannian manifolds admitting n linearly independent special concircular vector fields and investigate some relations between these vector fields and their associated scalar fields. In § 3, we prove that any Riemannian manifold admitting some linearly independent special concircular vector fields is an almost everywhere warped product, a part of which is a space of constant curvature, and obtain some results on the given manifold. Finally, in § 4, we shall give geometrical structures of Riemannian manifolds mentioned in § 3.

Throughout this paper, we assume that manifolds and quantities are differentiable of class C^∞.

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§ 1. Almost everywhere warped products

Let M_1 and M_2 be Riemannian manifolds of dimension m and $n-m$ respectively, and f a positive-valued differentiable function on M_1. The *warped product* $M=M_1\times{}_fM_2$ is by definition (see [1]) the product manifold $M_1\times M_2$ endowed with Riemannian metric

$$(X, X) = (\pi_1X, \pi_1X) + f^2(\pi_1x)(\pi_2X, \pi_2X)$$

for any vector $X\in T_x(M)$, $x\in M$, where π_α $(\alpha=1, 2)$ is the natural projection $M\to M_\alpha$, the tangential map of π_α is denoted by the same character, and (,) is the Riemannian inner product.