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Subgroup $(SU(2)\times Spin(12))/Z_2$ of compact simple Lie group E_7 and non-compact simple Lie group $E_{7,\sigma}$ of type $E_{7(-5)}$

Osami YASUKURA and Ichiro YOKOTA

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Introduction

It is known that there exist four simple Lie groups of type E_7 up to local isomorphism, one of them is compact and the others are non-compact. We have shown that in [3], [5] the group

$$\begin{aligned} E_7 &= \{\alpha \in \mathrm{Iso}_{\boldsymbol{C}}(\mathfrak{P}^{\boldsymbol{C}}, \mathfrak{P}^{\boldsymbol{C}}) \mid \alpha(P\times Q)\alpha^{-1} = \alpha P\times \alpha Q, \langle \alpha P, \alpha Q\rangle = \langle P, Q\rangle\} \\ &= \{\alpha \in \mathrm{Iso}_{\boldsymbol{C}}(P^{\boldsymbol{C}}, \mathfrak{P}^{\boldsymbol{C}}) \mid \alpha\mathfrak{M}^{\boldsymbol{C}} = \mathfrak{M}^{\boldsymbol{C}}, \{\alpha P, \alpha Q\} = \{P, Q\}, \langle \alpha P, \alpha Q\rangle = \langle P, Q\rangle\} \end{aligned}$$

is a simply connected compact simple Lie group of type E_7 and in [4], [5] the group

$$\begin{aligned} E_{7,\iota} &= \{\alpha \in \mathrm{Iso}_{\boldsymbol{C}}(\mathfrak{P}^{\boldsymbol{C}}, \mathfrak{P}^{\boldsymbol{C}}) \mid \alpha(P\times Q)\alpha^{-1} = \alpha P\times \alpha Q, \langle \alpha P, \alpha Q\rangle_\iota = \langle P, Q\rangle_\iota\} \\ &= \{\alpha \in \mathrm{Iso}_{\boldsymbol{C}}(\mathfrak{P}^{\boldsymbol{C}}, \mathfrak{P}^{\boldsymbol{C}}) \mid \alpha\mathfrak{M}^{\boldsymbol{C}} = \mathfrak{M}^{\boldsymbol{C}}, \{\alpha P, \alpha Q\} = \{P, Q\}, \langle \alpha P, \alpha Q\rangle_\iota = \langle P, Q\rangle_\iota\} \end{aligned}$$

is a connected non-compact simple Lie group of type $E_{7(-25)}$ and its polar decomposition is given by

$$E_{7,\iota} \simeq (U(1)\times E_6)/\boldsymbol{Z}_3 \times \boldsymbol{R}^{54}.$$

In this paper, we show that the group

$$\begin{aligned} E_{7,\sigma} &= \{\alpha \in \mathrm{Iso}_{\boldsymbol{C}}(\mathfrak{P}^{\boldsymbol{C}}, \mathfrak{P}^{\boldsymbol{C}}) \mid \alpha(P\times Q)\alpha^{-1} = \alpha P\times \alpha Q, \langle \alpha P, \alpha Q\rangle_\sigma = \langle P, Q\rangle_\sigma\} \\ &= \{\alpha \in \mathrm{Iso}_{\boldsymbol{C}}(\mathfrak{P}^{\boldsymbol{C}}, \mathfrak{P}^{\boldsymbol{C}}) \mid \alpha\mathfrak{M}^{\boldsymbol{C}} = \mathfrak{M}^{\boldsymbol{C}}, \{\alpha P, \alpha Q\} = \{P, Q\}, \langle \alpha P, \alpha Q\rangle_\sigma = \langle P, Q\rangle_\sigma\} \end{aligned}$$

is a connected non-compact simple Lie group of type $E_{7(-5)}$ with the center $z(E_{7,\sigma}) = \{1, -1\}$. The polar decomposition of the group $E_{7,\sigma}$ is given by

$$E_{7,\sigma} \simeq (SU(2)\times Spin\,(12))/\boldsymbol{Z}_2 \times \boldsymbol{R}^{64}.$$

To give this decomposition, we find subgroups

$$SU(2), \qquad Spin(12), \qquad (SU(2)\times Spin(12))/\boldsymbol{Z}_2$$

in the group E_7 and the group $E_{7,\sigma}$ explicitly.