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On the behavior of solutions of generalized Emden-Fowler equations with deviating arguments

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1. Introduction

In the last twenty years there has been considerable interest in the problem of classifying the nonoscillatory solutions of both ordinary and functional differential equations in terms of their asymptotic behavior. Much of the work in this direction has been the derivation of both necessary and sufficient conditions for the existence of certain types of nonoscillatory solutions. As examples of such results we cite the recent papers of Kusano and Onose [7] and Odarič and Ševelo [8], and the references contained therein.

Here we are concerned with the classification of the solutions of the n-th order functional differential equation

$$\begin{aligned}(r(t)x^{(n-\nu)}(t))^{(\nu)}\\ +\left(\textstyle\prod_{j=1}^{m}|x(g_j(t))|^{\rho_j}\right)(F(t,\, x^2(g_1(t)),\dots,\, x^2(g_m(t))))\textstyle\prod_{k=1}^{2q-1}\operatorname{sgn} x(g_{j_k}(t))=0\end{aligned} \tag{E}$$

using a classification scheme similar to the one employed in [7] and [8]. However our interest is in obtaining conditions on the functions r, F, and g_j which ensure that all nonoscillatory solutions of (E) belong to certain specified classes rather than the existence of such a solution in a given class as was done in [7] and [8]. While results of the same type as ours have been obtained by other authors, e.g. [3; Th. 5], our results differ in a number of ways from those previously obtained. For example, the form of our integral conditions (see (7), (8), (12), and (22) below) differ from those previously required by other authors; moreover, when $r(t)\not\equiv 1$, the fact that ν can be any integer satisfying $1\leq\nu\leq n-1$ allows for numerous combinations of middle terms in (E) not considered before.

Notice that equation (E) may be viewed as a generalization of the well known Emden-Fowler equation. For a discussion of the physical and historical significance of the latter equation the reader is referred to the excellent survey paper of Wong [11].

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