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The Hopf bifurcation and its stability for semilinear diffusion equations with time delay arising in ecology

Kiyoshi YOSHIDA

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Introduction

Time delay mechanism arises in the dynamics of one or several species. One of such models is

$$\dot{U} = d\Delta U + a(1-U(t-r)/k)U, \tag{0.1}$$

where U means a population density, a, d, K and r are positive constants and $\cdot = \partial/\partial t$. In the absence of diffusion, (0.1) is well known as Volterra-Hutchinson's equation (e.g. [11, p. 94]). It is not difficult to verify that there exists a global solution of (0.1) in $(0, \infty) \times \Omega$ with initial and homogeneous Neumann boundary conditions, where Ω is a bounded domain in $\boldsymbol{R}^n$ with the smooth boundary $\partial\Omega$ (see Proposition 1.1 below). From an ecologial point of view, this boundary condition describes the situation where some population is reserved in a domain surrounded by a reflecting wall. The work by Cohen and Rosenblat [5], Lin and Kahn [9], Murray [12] and Yamada [16] related to this field should be referred.

Our interest lies in the spatio-temporal fluctuation of population density around the spatially homogeneous and postitive steady state $u(t, x) \equiv K$ caused by time delays. For this problem we study that a spatially homogeneous and temprally periodic orbit bifurcates form $u \equiv K$ as the primary bifurcation when some parameter, say r, crosses a critical value. We also discuss here a stability of the bifurcating orbit. This is done by the approach due to Chow and Mallet-Paret [4].

From both ecological and mathematical viewpoints, it is interesting to consider time delay models which exhibit spatially inhomogeneous and temporally periodic orbits bifurcating from the trivial solution as the primary bifurcation. We will show such models in the forthcoming paper.

In this paper, taking the problem of a spatially inhomogeneous bifurcating orbit into consideration, we develop a basic theory, especially the construction of a local integral manifold, and show the existence of the Hopf bifurcation for (0.1) with the homogeneous Neumann boundary condition and its stability. Here we give a remark. Every solution of the ordinary functional differential equation (shortly, OFDE) corresponding to (0.1) (i.e., $d=0$) is a solution of (0.1)