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Modified Rosenbrock methods for stiff systems

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1. Introduction

Consider the initial value problem for a stiff system

$$(1.1)\qquad y' = f(y), \quad y(x_0) = y_0,$$

where y is an m-vector and the m-vector function $f(y)$ is assumed to be sufficiently smooth. Let $y(x)$ be the solution of this problem,

$$(1.2)\qquad x_j = x_0 + jh \qquad (j = 1, 2,\dots, h > 0),$$

and let $J(y)$ be the Jacobian matrix of $f(y)$. We are concerned with the case where approximations $y_j (j=1, 2,\dots)$ of $y(x_j)$ are computed by A-stable modified Rosenbrock methods of the form

$$(1.3)\qquad y_{n+1} = y_n + \textstyle\sum_{i=1}^{q} p_i k_i \qquad (n = 0, 1,\dots)$$

which require per step one evaluation of J, k evaluations of f and the solution of a system of m linear equations for q different right hand sides, where

$$(1.4)\qquad Mk_i = hf(y_n + \textstyle\sum_{j=1}^{i-1} a_{ij}k_j) + hJ \sum_{j=1}^{i-1} d_{ij}k_j \qquad (i = 1, 2,\dots, q),$$

the matrix $M = I - ahJ$ is nonsingular, $J = J(y_n + bhf(y_n))$, a and b are constants and $a > 0$.

Nørsett and Wolfbrandt [10] obtained an A-stable method of order 3 for $k=q=2$. Kaps and Rentrop [6] have constructed an A-stable method of order 4 which embeds a method of order 3 for $k=3$ and $q=4$. Kaps and Wanner [7] have shown that there exists no A-stable method of order $k+1$ for $k=q=4$, 5 and constructed an A-stable method of order k for $k=q=5$, 6.

Bui [2] derived an L-stable method of order k for $k=q=2$, 3, 4. Cash [4] has obtained a strongly A-stable method of order 3 which embeds a method of order 2 for $k=2$ and $q=4$. Artemev and Demidov [1] have proposed a variable order method which is A-stable and of order k for $k=1$, 2, 3, 4.

The first object of this paper is to show that for $q=2k+1$ $(k=1, 2, 3)$ we can construct an A-stable modified Rosenbrock method of order $k+2$ and also a method of order $k+1$ by incorporating the first value of f in the next step of integration. The discrepancy of these two methods can be used for stepsize