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Existence and periodicity of weak solutions of the Navier-Stokes equations in a time dependent domain

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Introduction

This paper deals with the problem of existence and periodicity of weak solutions of the initial-boundary value problem for the Navier-Stokes equations in domains with smoothly moving boundaries. Hopf [5] proved the existence of a global weak solution in a cylindrical domain by using the Faedo-Galerkin approximation. On the other hand, Fujita-Sauer [4] and Lions [8] obtained the same result in the case of time dependent domains with Lipschitz continuous boundaries by a penalty method ([4]) or a singular perturbation method ([8]). Our main purpose in this paper is to show that the method of Hopf [5] can be applied with a slight modification to the case when the domain moves smoothly. An advantage of Hopf's method is that we can show the existence of a periodic solution when the domain moves periodically and the boundary data are small enough.

To show the existence of a weak solution we reduce in Section 1 the given problem to the one in a cylindrical domain, assuming the existence of a diffeomorphism which sends the given time dependent domain to a cylindrical one. In doing so, the velocity and the pressure gradient will be transformed as vector fields. Similar techniques are used in Bock [1] and in Inoue-Wakimoto [6], where the existence of a unique local strong solution is proved by the Faedo-Galerkin method ([1]) or the method of evolution equation in Hilbert space ([6]). However, Bock [1] does not regard the velocity and the pressure gradient as vector fields, and so the calculation given in [1] is complicated. In [6] Inoue and Wakimoto treat the velocity and the pressure gradient as vector fields, but they assume that the Jacobian of the diffeomorphism is equal to 1, which is a strong limitation. In this paper we assume that the Jacobian of the diffeomorphism depends only on time variable. As will be shown in Section 4, this assumption for the Jacobian is of no restriction.

Section 2 deals with the construction and estimate of approxiamte solutions. We first construct approximate solutions for the reduced problem by choosing a suitable Galerkin basis, and then return to the original problem on a time dependent domain to get an energy inequality, which together with a modification of the compactness argument given in [5] enables us to choose a subsequence of