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Generalized hypergeometric equations of non-Fuchsian type

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1. Introduction

We consider a connection problem for the differential equation

$$z^n y^{(n)} = \sum_{l=1}^{n} (a_l + b_l z^q) z^{n-l} y^{(n-l)}, \tag{1.1}$$

where q is a complex number and the a_l, b_l are complex constants. This differential equation has only two singularities at the origin and infinity in the whole complex z-plane, and so, it may be assumed without loss of generality that $\operatorname{Re} q \geqq 0$. In [3] we dealt with a case in which $b_l = 0$ $(l = 0, 1, \dots, n-1)$. In this case (1.1) is of just the extended form of the classical Bessel equation. By solving the connection problem for it and investigating global behaviors of such solutions, we could obtain the extension of the Airy function and the Bessel function. In this paper we shall treat a general case in which $b_l = 0$ $(l = 1, 2, \dots, \nu - 1)$ and $b_\nu \neq 0$ for $1 \leqq \nu < n$. As is well-known, (1.1) can be reduced to a generalized hypergeometric equation. In fact, let us denote

$$[\rho]_n - \sum_{l=1}^{n} a_l [\rho]_{n-l} = \prod_{j=1}^{n} (\rho - \hat{\rho}_j),$$
$$\sum_{l=\nu}^{n} b_l [\gamma]_{n-l} = b_\nu \prod_{j=1}^{n-\nu} (\gamma - \hat{\gamma}_j),$$

where brackets imply the Pochhammer notation, i.e.,

$$[\rho]_p = \rho(\rho - 1) \cdots (\rho - p + 1), \quad [\rho]_0 = 1.$$

Then (1.1) can be written in the form

$$\left[\prod_{j=1}^{n} (D - \hat{\rho}_j)\right] y = b_\nu z^q \left[\prod_{j=1}^{n-\nu} (D - \hat{\gamma}_j)\right] y \qquad (D = z d/dz).$$

The change of variables $z = t^\alpha$ yields

$$\left[\prod_{j=1}^{n} (\mathscr{D} - \alpha \hat{\rho}_j)\right] y = b_\nu \alpha^\nu t^{q\alpha} \left[\prod_{j=1}^{n-\nu} (\mathscr{D} - \alpha \hat{\gamma}_j)\right] y \qquad (\mathscr{D} = t d/dt).$$

Putting

$$\alpha = \nu/q, \quad b_\nu \alpha^\nu = \beta,$$
$$\rho_j = \alpha \hat{\rho}_j \quad (j = 1, 2, \dots, n), \ \gamma_j = \alpha \hat{\gamma}_j \quad (j = 1, 2, \dots, n - \nu),$$

we thus obtain a general form of non-Fuchsian generalized hypergeometric equation