HIROSHIMA MATH. J.
13 (1983), 65–81

Global existence of solutions to nonautonomous differential equations in Banach spaces

Toshiyuki IWAMIYA

(Received May 10, 1982)

1. Introduction

Let X be a Banach space over the real field $\boldsymbol{R}$. Let Ω be a subset of $[a, b) \times X (a < b \leqq \infty)$ and A a continuous function from Ω into X. In this paper we study the initial-value problem for a nonautonomous differential equation in X

$$(1.1) \qquad u' = A(t, u), \qquad u(\tau) = z,$$

where (τ, z) is given in Ω.

This problem has been studied by many authors and the present paper is related to the works of Crandall [1], Deimling [2], Kato [3], [4], Kenmochi and Takahashi [5], Lakshmikantham, Mitchell and Mitchell [7], Lovelady and Martin [8], Martin [9], [10], and Pavel and Vrabie [11]. In the works [1], [7] and [9] the problem is treated in cace of cylindrical domain Ω (i.e., Ω is of the form $[a, b) \times D$); and in [5], [4], Kenmochi and Takahashi, and then Kato, generalized the results as obtained in the works [7] and [9] to allow the Ω to be genuinely noncylindrical.

Our purpose of this paper is to establish an existence and uniqueness theorem for solutions of (1.1) under three general conditions (called herein (Ω2), (Ω3) and (Ω4)) in addition to the condition (Ω1) that A is continuous. Although precise statements of these conditions are given in Section 2, we here make some mention of them in order to illustrate features of this paper. Condition (Ω2) imposes on the domain Ω a closedness condition in a certain sense. For instance, if $\Omega = [a, b) \times D$ and D is a closed subset of X then condition (Ω2) is satisfied. Condition (Ω3) is the so-called subtangential condition (cf. [5]) which is utilized to construct approximate solutions for (1.1). Condition (Ω4) is a relaxation of dissipativity conditions as employed in the papers cited above and guarantees the unicity of solutions to (1.1). Accordingly, condition (Ω4) generalizes most of conditions which are usually treated in the theory of ordinary differential equations. Under these conditions, we first investigate the local existence of solutions to (1.1). We then give a global existence theorem via the local existence result as well as the continuous dependence of solutions on initial data. Our result on the global existence is obtained under general conditions as mentioned above. Hence it extends most of the known results concerning the global existence of solutions of nonautonomous equations of the form (1.1).