HIROSHIMA MATH. J
13 (1983), 39–51

On the asymptotic properties for simple semilinear heat equations

KUSUO KOBAYASHI

(Received May 20, 1982)

§1. Introduction

It is known that solutions of Cauchy problems for some semilinear evolution equations may blow up in a finite time (or grow up to infinity as $t\to\infty$) for some initial values. There are several works concerning the asymptotic behavior of the solution of the Cauchy problem for the equation

$$\frac{\partial}{\partial t}u(t, x) = \Delta u(t, x) + g(u(t, x)), \quad t > 0,\ x \in \boldsymbol{R}^N, \tag{1.1}$$

with the initial condition

$$u(0, x) = a(x), \quad x \in \boldsymbol{R}^N. \tag{1.2}$$

The case when $g(\lambda)=\lambda^{1+\alpha}$ $(\alpha>0)$ has been studied by H. Fujita [1], [2], K. Hayakawa [3] and S. Sugitani [7]. Assume that the initial value $a(x)$ is non-negative bounded continuous. Then these results can be stated as follows;

(i) in case $0<\alpha N\leq 2$, for any initial value $a(x)$ not vanishing identically, the solution $u(t, x)$ of (1.1) with (1.2) blows up in a finite time, and

(ii) in case $\alpha N>2$, (a) for sufficiently small initial values $a(x)$ $(\not\equiv 0)$ the solutions $u(t, x)$ of (1.1) with (1.2) converge to 0 uniformly in x as $t\to\infty$, and (b) for sufficiently large initial values $a(x)$ the solutions $u(t, x)$ of (1.1) with (1.2) blow up in a finite time.

For general f, there is a work of K. Kobayashi-T. Sirao-H. Tanaka [5].

Under what condition on the initial value $a(x)$ does the solution $u(t, x)$ of (1.1) with (1.2) converge to 0 as $t\to\infty$ in case $\alpha N>2$? And, under what condition on $a(x)$ does the solution $u(t, x)$ blow up in a finite time in the same case?

In this paper we shall consider these kinds of problems for the equation (1.1) replacing g by f defined as follows:

$$f(\lambda) = \begin{cases} p\lambda - pq, & \lambda \geq q, \\ 0, & 0 \leq \lambda < q, \end{cases} \tag{1.3}$$

where p and q are positive constants.

For any bounded continuous function $a(x)$ on $\boldsymbol{R}^N$, it is known that the equa-