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Lie algebras in which the join of any set of subideals is a subideal

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Introduction

In the recent paper [2], the author investigated several classes of Lie algebras in which the join of any pair of subideals (resp. weak subideals) is always a subideal (resp. a weak subideal). The purpose of this paper is to present further resutls concerning the class of Lie algebras in which the join of any set of subideals (resp. weak subideals) is always a subideal (resp. a weak subideal).

In Section 1 we shall characterize the class $\mathfrak{L}^{\infty}$ (resp. $\overline{\mathfrak{L}}^{\infty}$) of Lie algebras in which the join of any set of subideals (resp. weak subideals) is always a subideal (resp. a weak subideal), that is, $\mathfrak{L} \cap (\mathfrak{L}^{\infty}$ Max-si$) = \mathfrak{L}^{\infty}$ and $\overline{\mathfrak{L}} \cap (\overline{\mathfrak{L}}^{\infty}$ Max-wsi$) = \overline{\mathfrak{L}}^{\infty}$ (Theorem 1). In Section 2 we shall show that over fields of characteristic zero, $\mathfrak{E} \cap (\mathfrak{L}^{\infty}$ Max-si$) \leq \mathfrak{L}^{\infty}$ (Theorem 7). As concrete subclasses of $\mathfrak{L}^{\infty}$ of Lie algebras over a field of characteristic zero, we shall present $\mathfrak{N}$ Max-si and $\mathfrak{E} \cap$ (($\mathfrak{N}$ Max-si) Max-si) (Theorem 8). In Section 3 we shall show that over fields of characteristic zero, $(\mathfrak{F} \cap \mathfrak{N})$ (Min-si $\cap$ Max-si) $\leq \mathfrak{L}^{\infty} \cap \mathfrak{L}_{\infty}$ (Theorem 10).

Throughout the paper we employ the notations and terminology in [1] and [2], and all Lie algebras are over a field of arbitrary characteristic unless otherwise specified.

1.

Tôgô [4] introduced the class Min-wsi of Lie algebras satisfying the minimal condition for weak subideals. We analogously introduce the class Max-wsi of Lie algebras satisfying the maximal condition for weak subideals. On the other hand, we introduced the class $\mathfrak{L}$ (resp. $\mathfrak{L}^{\infty}$) [2] of Lie algebras in which the join of any pair (resp. any set) of subideals is always a subideal. We similarly introduced the class $\overline{\mathfrak{L}}$ (resp. $\overline{\mathfrak{L}}^{\infty}$) for the case of weak subideals. We shall first show the following

THEOREM 1. (1) $\mathfrak{L} \cap (\mathfrak{L}^{\infty}$ Max-si$) = \mathfrak{L}^{\infty}$.
(2) $\overline{\mathfrak{L}} \cap (\overline{\mathfrak{L}}^{\infty}$ Max-wsi$) = \overline{\mathfrak{L}}^{\infty}$.

PROOF. Here we only prove (1), since (2) is proved similarly. Clearly we have $\mathfrak{L}^{\infty} \leq \mathfrak{L} \cap (\mathfrak{L}^{\infty}$ Max-si$)$. Conversely, let $L \in \mathfrak{L} \cap (\mathfrak{L}^{\infty}$ Max-si$)$ and let $\{H_{\alpha}$: