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Invariant measures for uniformly recurrent diffusion kernels

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In this paper we shall characterize invariant measures for a uniformly recurrent diffusion kernel T on a locally compact Hausdorff space X. Our main result is summarized as follows: Denote by $H(T)$ the cone generated by non-negative T-invariant measures and put $X_o = \mathrm{cl}\,(\cup_{\mu \in H(T)} \mathrm{supp}\,(\mu))$. Then there exists a strictly positive diffusion kernel W on X_o, uniquely determined except for the equivalence of diffusion kernels, such that $TW = W$ and $H(T)$ coincides with W-potentials.

In sections 2 and 3, we shall discuss when $H(T)$ is one dimensional and when the cone formed by non-negative invariant functions with respect to the transposed kernel of T is one dimensional.

We remark in section 4 that similar results are valid for uniformly recurrent continuous diffusion semi-groups on X.

A typical example of a uniformly recurrent diffusion kernels is an idempotent kernel on X. Applying our theorem to the idempotent kernels and using results in M. Itô [10], we see that a weakly regular diffusion kernel on X may be considered as a weakly regular Hunt diffusion kernel on some quotient space of X.

In section 6, applying our theorem to diffusion kernels of convolution type on homogeneous spaces, we represent explicitly the above diffusion kernel W. In this direction, for a locally compact abelian group G and non-negative adapted Radon measure σ on G, G. Choquet and J. Deny [4] showed that all extreme rays of the convex cone $H(\sigma)$ formed by non-negative σ-invariant measures are generated by exponentials on G. In a non-abelian case, H. Furstenberg [6] pointed out that the extreme rays of $H(\sigma)$ are generated by multiplier functions on a certain Lie group G and some particular measure σ; however a caracterization of the extreme rays is not known in the general case. But if σ is recurrent, our theorem shows that $H(\sigma)$ is generated by at most one exponential on G even if G is not commutative (see also [7]). Using our theorem, we can characterize non-negative finite order measures on locally compact Hausdorff groups, particularly, we see that non-negative idempotent measures are the normalized Haar measures (cf. [9] and [13]).

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