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On the map defined by regarding embeddings as immersions

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Introduction

Let M be a closed connected smooth manifold of dimension n and R^m the m-dimensional Euclidean space. Denote by $[M \subseteq R^m]$ the set of regular homotopy classes of immersions of M in R^m and by $[M \subset R^m]$ the set of isotopy classes of embeddings of M in R^m, and consider the commutative diagram

$$\begin{array}{ccc} [M \subset R^{m+1}] & \xrightarrow{J_{m+1}} & [M \subseteq R^{m+1}] \\ E_m \uparrow & & I_m \uparrow \\ [M \subset R^m] & \xrightarrow{J_m} & [M \subseteq R^m], \end{array}$$

where E_m and I_m are the maps induced from the natural inclusion $R^m \subset R^{m+1}$ and J_k is the one defined by regarding embeddings as immersions.

The set $[M \subseteq R^m]$ for $2m > 3n+1$ is an abelian group by taking 0 arbitrarily if it is not empty, and the map I_m is a homomorphism by taking $I_m(0)=0$; while so are the set $[M \subset R^m]$ and the maps E_m and J_m for $2m > 3(n+1)$ (see J. C. Becker [2]).

The purpose of this paper is to study the above commutative diagram when $m = 2n-1$:

$$(*) \qquad \begin{array}{ccc} [M \subset R^{2n}] & \xrightarrow{J_{2n}} & [M \subseteq R^{2n}] \\ E \uparrow & & I \uparrow \\ [M \subset R^{2n-1}] & \xrightarrow{J_{2n-1}} & [M \subseteq R^{2n-1}] \end{array} \qquad (E = E_{2n-1},\ I = I_{2n-1}),$$

(here we assume that the sets in consideration are not empty).

When $n \geqq 4$, the upper groups are determined by A. Haefliger and M. W. Hirsch [3], [5], [6] and so is the group $[M \subseteq R^{2n-1}]$ by D. R. Bausum [1, Th. 37 and Prop. 41], L. L. Larmore and E. Thomas [10, Th. 5.1] and R. D. Rigdon [11, Th. 10.4], and moreover it is proved by R. D. Rigdon [11, Th. 10.4] that I is trivial for even n and is surjective for odd n, respectively. When $n \geqq 6$, $[M \subset R^{2n-1}]$ is an abelian group and Im E is determined by R. D. Rigdon [11, Th. 11.11 and Th. 11.26]. Together with these results, we have the following

MAIN THEOREM. *Let M be a closed connected smooth manifold of dimension*