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A note on bounded positive entire solutions of semilinear elliptic equations

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In this note we are concerned with bounded positive entire solutions of the second order semilinear elliptic equation

$$\Delta u + a(x)f(u) = 0, \quad x \in R^n, \tag{1}$$

where $n \geqq 3$ and Δ is the n-dimensional Laplace operator. By an entire solution of (1) we mean a function $u \in C^2(R^n)$ which satisfies (1) at every point of R^n. We assume throughout that $a(x)$ is a locally Hölder continuous function on R^n and $f(u)$ is a locally Lipschitz continuous function on $(0, \infty)$ which is positive and nondecreasing for $u > 0$. As usual, $|x|$ denotes the Euclidean length of $x \in R^n$.

Our result is the following:

THEOREM. *Suppose that there exist locally Hölder continuous functions $a_*(t)$ and $a^*(t)$ on $[0, \infty)$ such that*

$$-a_*(|x|) \leqq a(x) \leqq a^*(|x|) \quad \textit{for} \quad x \in R^n; \tag{2}$$

$$a_*(t) \textit{ and } a^*(t) \textit{ are nonnegative for } t \geqq 0; \tag{3}$$

$$\int_0^\infty t a_*(t)dt = A_* < \infty \quad \textit{and} \quad \int_0^\infty t a^*(t)dt = A^* < \infty. \tag{4}$$

Define the sets L_* and L^* by

$$L_* = \{\ell \mid \ell > 0 \textit{ and } \ell - f(\ell)A_*(n-2)^{-1} > 0\}, \tag{5}$$

$$L^* = \{\ell \mid \ell = c - f(c)A^*(n-2)^{-1} > 0 \textit{ for some } c > 0\}, \tag{6}$$

and suppose that $L_* \cap L^*$ is nonempty.

Then, for any $\ell \in L_* \cap L^*$, there exists an entire solution $u(x)$ of (1) *which is positive for $x \in R^n$ and satisfies*

$$u(x) \longrightarrow \ell \quad \textit{as} \quad |x| \longrightarrow \infty. \tag{7}$$

Observe that, in the case of $f(u) = u^\gamma$, if $A_* = A^* > 0$ then the set $L_* \cap L^*$ becomes the interval:

$$L_* \cap L^* = (0, (1-\gamma^{-1})((n-2)/\gamma A^*)^{1/(\gamma-1)}] \qquad \text{for} \quad \gamma > 1;$$