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Asymptotic theory of perturbed general disconjugate equations II

William F. TRENCH

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1. Introduction

Here we continue the investigation begun in [9] of the asymptotic behavior of solutions of the equation

$$L_n u + Fu = 0, \tag{1}$$

where L_n is the general disconjugate operator

$$L_n = \frac{1}{p_n}\frac{d}{dt}\frac{1}{p_{n-1}}\cdots\frac{1}{p_1}\frac{d}{dt}\frac{\cdot}{p_0} \qquad (n \geqq 2), \tag{2}$$

with $p_i > 0$ and $p_i \in C[a, \infty)$, $0 \leqq i \leqq n$. Although we did not make specific assumptions in [9] on the form of the functional F in (1), we restrict our attention here to the case where (1) is of the form

$$L_n u + F(t, L_0 u, \dots, L_{n-1} u) = 0, \tag{3}$$

with

$$L_0 x = \frac{x}{p_0}; \quad L_r x = \frac{1}{p_r}(L_{r-1}x)', \quad 1 \leqq r \leqq n. \tag{4}$$

Nevertheless, for convenience we abbreviate (3) as in (1), and write

$$F(t, L_0 u(t), \dots, L_{n-1} u(t)) = (Fu)(t).$$

We say that u is a solution of (3) if $L_0 u, \dots, L_n u$ exist and satisfy (3) on a half line $[t_0, \infty)$ for some $t_0 \geqq a$. We seek conditions which imply that (3) has a solution u which behaves for large positive t like a given solution q of the unperturbed equation

$$L_n x = 0, \tag{5}$$

in a sense defined below. We believe that our results are new even in the case where

$$p_0 = p_1 = \cdots = p_n = 1, \tag{6}$$