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Self H-equivalences of H-spaces with applications to H-spaces of rank 2

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Introduction

The homotopy classification of spaces and maps is a subject of classical studies in algebraic topology. The group $\mathscr{E}(X)$ of self equivalences of a space X and the subgroup $\mathscr{E}_H(X)$ of self H-equivalences of an H-space X arose from such classification problem. For a based space X, $\mathscr{E}(X)$ is defined to be the set of all homotopy classes of homotopy equivalences of X to itself with group multiplication induced by the composition of maps; and it has been investigated by several authors including [2], [10], [19], [20] and [22], where calculating $\mathscr{E}(X)$ has been made with two exact sequences, originally due to Barcus-Barratt [2], given by either the skeletons or the Postnikov system of X. When X is an H-space, $\mathscr{E}_H(X)$ is defined to be the subgroup of $\mathscr{E}(X)$ consisting of H-maps, which has been studied in [13] and [24] for instance. But much less examples of calculation are known; in fact, when X is a finite 1-connected H-complex (H-space being a CW-complex), $\mathscr{E}_H(X)$ has determined only in case that X is of rank$\leqq 2$ with no torsion in homology.

This paper is divided into two parts. In Part I, we present an exact sequence for calculating $\mathscr{E}_H(X)$ of a 1-connected H-complex X in terms of its Postnikov system. The aim of Part II is the determination of $\mathscr{E}_H(G_{2,b})$ made use of the exact sequence given in Part I, where $G_{2,b}$ $(-2\leqq b\leqq 5)$ are of rank 2 with torsion in homology given by Mimura-Nishida-Toda [17].

Let X be a 1-connected H-complex, and consider the Postnikov system $\{X_n\}$ of X with obvious map f_n: $X\to X_n$ and usual fiber sequence

(1) $$\Omega X_{n-1} \xrightarrow{\Omega k} K(\pi_n, n) \xrightarrow{i_n} X_n \xrightarrow{p_n} X_{n-1} \xrightarrow{k} K(\pi_n, n+1)$$

(Ω is the loop functor)

where $\pi_n(X)$ is sometimes abbreviated to π_n and the Postnikov invariant k^{n+1} to k. Then, the theorem of J. D. Stasheff [26, Th. 5] states that X_n is an H-space in such a way that all the structure maps f_n, k, p_n and i_n are H-maps; and we have proved in the previous paper [25, Th. 1.3] that

(2) f_n *induces a homomorphism* $f_{n!}$: $\mathscr{E}_H(X)\to\mathscr{E}_H(X_n)$ *which is monomorphic if* $n\geqq\dim X$ *and isomorphic if* $n\geqq 2\dim X$.