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Martin boundary for $\Delta - P$

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§1. One of interesting problems concerning the Martin boundary is to determine when it is homeomorphic to the Euclidean boundary. In the present note, we give sufficient conditions for the Martin boundary of a bounded domain Ω in the n-dimensional Euclidean space $\boldsymbol{R}^n$ ($n \geqq 2$) with respect to the operator of the form $L_P = \Delta - P$ to be homeomorphic to the Euclidean boundary $\partial\Omega$, where Δ denotes the Laplace operator and P is a non-negative locally Hölder continuous function on Ω.

We say that a domain Ω has *bounded curvature* if there exists a positive number d such that for each point $Y \in \partial\Omega$ there exist points x_y and x'_y such that

$$B(x_y, d) \subset \Omega,\ B(x'_y, d) \subset C\bar{\Omega} \quad \text{and} \quad Y \in \partial B(x_y, d) \cap \partial B(x'_y, d),$$

where $B(x, d)$ denotes the open ball in $\boldsymbol{R}^n$ of radius $d > 0$ centered at x. We call d an *admissible radius* of Ω. We define a function δ_Ω on Ω by

$$\delta_\Omega(x) = \operatorname{dist}(x, \partial\Omega).$$

Our main result is the following

THEOREM 1. *Let Ω be a bounded domain in $\boldsymbol{R}^n$ of bounded curvature. Suppose that a non-negative locally Hölder continuous function P on Ω satisfies the following condition* (*a*) *or* (*b*):

(*a*) *Ω is of $C^{1,\alpha}$-class* ($\alpha > 0$) *and*

$$\int_0 r\{\max_{\delta_\Omega(x) \geqq r} P(x)\} dr < \infty.$$

(*b*) *$P \in L^q(\Omega)$ for some $q > n/2$.*

Then the Martin boundary of Ω with respect to $L_P = \Delta - P$ is homeomorphic to the Euclidean boundary.

Related results have been given by A. Ancona ([1], Théorème 6) and H. Imai ([5], Theorem 2). Ancona showed the equivalence of the Martin boundary and the Euclidean boundary of a bounded Lipschitz domain in the half-space $\boldsymbol{R}^n_+ = \{x \in \boldsymbol{R}^n;\ x_n > 0\}$ with respect to an elliptic operator L of the form

$$Lu(x) = \sum a_{ij}(x)u_{ij}(x) + x_n^{-1} \sum b_i(x)u_i(x) + x_n^{\beta-2}c(x)u(x),$$