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Self-adjoint harmonic spaces and Dirichlet forms

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1. Introduction and notations

The aim of this paper is to clarify the relation between energy forms on a self-adjoint harmonic space $(X, \mathscr{H})$ studied by Maeda in [7] (cf. also [6]) and Dirichlet forms on $L^2(X; m)$ in the sense of Fukushima [5] and Silverstein [11]. Here X denotes a locally compact Hausdorff space with a countable base, connected and locally connected, $\mathscr{H}$ the harmonic sheaf and m a positive Radon measure on X. More precisely: we determine the set of all positive Radon measures m on X such that Maeda's energy form E with domain $\mathscr{E}_0$ can be considered as an „extended Dirichlet space with reference measure m" as defined in [5] und [11].

Let us recall the basic definitions and notations and give a brief review of Maeda's construction of energy forms.

Let $(X, \mathscr{H})$ be a self-adjoint harmonic space as defined in [6] §1.2. In particular we assume that the constant function 1 is superharmonic (Axiom 4 in [6]). Let G denote the symmetric (up to a multiplicative constant unique) Green function of X. Let ${}^*\mathscr{H}^+(X)$ denote the set of all positive hyperharmonic functions on X. $(X, {}^*\mathscr{H}^+(X))$ is a standard balayage space in the sense of [2]. Let τ_f denote the ${}^*\mathscr{H}^+(X)$-fine topology on X; i.e., the coarsest topology on X such that each function in ${}^*\mathscr{H}^+(X)$ is continuous with respect to τ_f. Notations with respect to τ_f will be designated by the prefix „fine(ly)-". For a numerical function g on X let $\hat{g}$ denote the greatest lower semi-continuous minorant of g. Define for $u \in {}^*\mathscr{H}^+(X)$ and $A \subset X$

$$R_u^A := \inf\{v \in {}^*\mathscr{H}^+(X): v \geq u \quad \text{on} \quad A\},$$

then $\hat{R}_u^A$ is the so-called balayage of u on A. Let $\mathscr{M}$ denote the set of all Radon measures on X and $\mathscr{M}^+ := \{\mu \in \mathscr{M}: \mu \geq 0\}$. We define for $\mu \in \mathscr{M}^+$

$$G\mu := \int G(\cdot, y) d\mu(y)$$

and for $\mu \in \mathscr{M}$ and $x \in \{G\mu^+ < \infty\} \cup \{G\mu^- < \infty\}$

$$G\mu(x) = G\mu^+(x) - G\mu^-(x),$$