Hiroshima Math. J.
14 (1984), 425–449

Coexistence problem for two competing species models with density-dependent diffusion

Masayasu Mimura†, Yasumasa Nishiura††, Alberto Tesei††† and Tohru Tsujikawa†

(Received January 18, 1984)

Abstract. We study the pattern formation of the Gause-Lotka-Volterra system of competition and nonlinear diffusion. This problem is related to segregation patterns between two competing species. It is shown that coexistence is possible by the effect of cross-population pressure in the situation where the inter-specific competition is stronger than the intra-specific one.

1. Introduction

In recent years, reaction-diffusion equation models have been proposed for the study of population dynamics. Shigesada et al. [17] proposed a one dimensinonal model of two competing species with self- and cross-population pressures

$$(1.1)\qquad \begin{aligned} u_t &= [(d_{11}+d_{12}v)u]_{xx} + (r_1 - a_{11}u - a_{12}v)u, \\ v_t &= [(d_{22}+d_{21}u)v]_{xx} + (r_2 - a_{21}u - a_{22}v)v, \end{aligned}$$

where u, v are the population densities of the two competing species, d_{ii} and d_{ij} $(i\neq j)$ are the self- and cross-diffusion rates, r_i are the intrinsic growth rates, a_{ii} and a_{ij} $(i\neq j)$ are the intra- and interspecific coefficients of competition. If $d_{ij}=0$ $(i\neq j)$, (1.1) is reduced to a normal competition-diffusion equation that has been extensively investigated. Kishimoto [9] proved that any nonnegative non-constant steady state solutions are unstable under zero flux boundary conditions. This result ecologically interprets that there occurs no spatial segregation between two competing species. On the other hand, Mimura and Kawasaki [12] showed that for suitable $d_{12}>0$ and/or $d_{21}>0$ there exist new non-constant steady state solutions bifurcating from a trivial solution

$$(\bar{u}, \bar{v}) = \left(\frac{r_1a_{22}-r_2a_{12}}{a_{11}a_{22}-a_{12}a_{21}}, \frac{r_2a_{11}-r_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}\right)$$

when r_i, a_{ij} $(i, j=1, 2)$ are chosen to satisfy $a_{12}/a_{22}<r_1/r_2<a_{11}/a_{21}$. This occurs on the basis of the cross-diffusion induced instability.

From an ecological point of view, it is quite interesting to study coexistence problem under $a_{11}/a_{21}<a_{12}/a_{22}$. The reason is that when $d_{ij}=0$ $(i\neq j)$, (1.1) never exhibit coexistence of two species and only one species can survive in com-