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Locally finite simple Lie algebras

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In the study of infinite-dimensional Lie algebras, the notions of ascendant subalgebras and serial subalgebras are fundamental. The notions generalizing these ones, weakly ascendant subalgebras and weakly serial subalgebras, were introduced and investigated in [7] and [2]. On the other hand, taking account of a result of Levič [5], a recent result of Stewart [6, Theorem 8] is expressed as follows: A locally finite Lie algebra over a field of characteristic 0 has no non-trivial ascendant subalgebras if and only if it has no non-trivial serial subalgebras.

In connection with these, we shall mainly study locally finite simple Lie algebras over a field $\mathfrak{k}$ of arbitrary characteristic. Actually there exist locally finite simple infinite-dimensional Lie algebras (Example 3).

In Section 2, we shall show that for a locally finite Lie algebra L over $\mathfrak{k}$, if H wser L then $H/\mathrm{Core}_L(H)$ is locally nilpotent (Theorem 5). We shall use this to give a simple proof and a refinement of Stewart's result stated above (Theorem 7).

In Section 3, we shall show that for a locally finite non-abelian simple Lie algebra L over $\mathfrak{k}$, if H wser L and $H \neq L$ then any finite-dimensional subalgebra of H belongs to $\mathfrak{e}^*(L)$, and

$$\begin{aligned} &\cup \{H \mid H \text{ wser } L,\ H \neq L\} = \cup \{H \mid H \text{ wasc } L,\ H \neq L\} \\ &= \cup \{H \mid H \leq^{\omega} L,\ H \neq L\} = \cup \{H \mid H \leq L,\ H \in \mathfrak{e}^*(L)\} = \mathfrak{e}(L) \end{aligned}$$

(Theorem 10). As a consequence of this we shall show that a locally finite non-abelian Lie algebra L over $\mathfrak{k}$ has no non-trivial weakly ascendant subalgebras if and only if L has no non-trivial weakly serial subalgebras, if and only if L is simple with $\mathfrak{e}^*(L) = \{0\}$, and if and only if L is simple with $\mathfrak{e}(L) = 0$ (Theorem 11).

1.

Throughout this paper, $\mathfrak{k}$ is a field of arbitrary characteristic unless otherwise specified, and L is a not necessarily finite-dimensional Lie algebra over $\mathfrak{k}$. When H is a subalgebra (resp. an ideal) of L, we denote $H \leq L$ (resp. $H \lhd L$).

Let $H \leq L$. For an ordinal ρ, H is a ρ-step weakly ascendant subalgebra (resp. a ρ-step ascendant subalgebra) of L, denoted by $H \leq^{\rho} L$ (resp. $H \lhd^{\rho} L$), if