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The Riemann-Hurwitz relation, parallel slit covering map, and continuation of an open Riemann surface of finite genus

Dedicated to Prof. M. Ohtsuka on his 60$^{\text{th}}$ birthday

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Introduction

Riemann proved, in 1851, the famous mapping theorem which is now named after him. It is the fountainhead of the study of conformal mapping. As its generalization, Schottky [32] suggested in his thesis that every finitely connected plane domain is mapped conformally onto a parallel slit plane. The first complete proof of this fact was due to Cecioni [4] and Hilbert [12]. Hilbert also outlined a proof for the case of infinite connectivity. Courant and Koebe carried out Hilbert's plan and, in fact, they finally showed that an arbitrary planar (=schlichtartig) Riemann surface can be mapped conformally onto a parallel slit plane. The mapping function is furnished by a "Strömungsfunktion" which is derived from a dipole "Strömungspotential" (see [14], p. 454 and p. 484).

In 1950, Nehari [23] first succeeded in generalizing the above result to (the interior of) compact bordered Riemann surfaces. Later Kusunoki [18] proved the same theorem again as an application of his theory of Abelian integrals on open Riemann surfaces. Mori [21] and Mizumoto [20] dealt with the general case — surfaces of finite genus but with infinitely many ideal boundary components.

Every author mentioned above at first constructed a single-valued meromorphic function on the surface which gives rise to a parallel slit (covering) mapping. Such a function is immediately recognized as a natural generalization of a "Strömungsfunktion". For this reason, we shall refer to it as an *S-function*. The existence of a non-constant S-function on an arbitrary surface of finite genus is assured by *e.g.*, the Riemann-Roch theorem (on open surfaces). It is known that every non-constant S-function defines a finite-sheeted covering surface of the extended complex plane $\hat{C}$.

Mizumoto's work as well as Mori's concerning the general case left some important problems open, however. The geometric structure of the covering determined by an S-function has not been fully analyzed. For instance, they asserted nothing about the branch points. In the beginning of this paper we