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On a product formula for a class of nonlinear evolution equations

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§ 0. Introduction

This work is concerned with the initial value problem of the form

$$\text{(IVP; } u_0\text{)} \qquad (d/dt)u(t) + \partial\phi u(t) \ni Bu(t), \quad t > 0,$$
$$u(0) = u_0,$$

where φ is a proper lower semicontinuous (l.s.c.) convex functional on an abstract real Hilbert space $\mathbf{H}$, $\partial\varphi$ is its subdifferential and B is a single-valued operator in $\mathbf{H}$ with domain $D(B)$ containing the effective domain $D(\varphi)$ of φ. Initial value problems of this type have been studied by many authors (e.g. [**5**, **11**, **12**, **13**]).

Let $\{S(\tau); 0 \leq \tau < \infty\}$ be the nonlinear contraction semigroup on $\overline{D(\varphi)}$ generated by $-\partial\varphi$, and $\{V(\tau); 0 \leq \tau < \infty\}$ a one-parameter family of single-valued operators $V(\tau)$ in $\mathbf{H}$ with $D(V(\tau)) \supset D(B)$ such that $\tau^{-1}(V(\tau)-1) \to B$ as $\tau \downarrow 0$ in a certain sense. (However the family $\{V(\tau)\}$ is not assumed to be a contraction semigroup on $\overline{D(B)}$.) In this paper it is our main interest to establish an existence theorem for (IVP; u_0) by showing that

$$\text{(0.1)} \qquad u_n(t) = [V(\tau(n))S(\tau(n))P]^{[t/\tau(n)]}u_0 \longrightarrow u(t) \quad \text{in} \quad \mathbf{H} \quad \text{as} \quad n \to \infty$$

and

$$\varphi(S(\tau(n))Pu_n(t)) \longrightarrow \varphi(u(t)) \quad \text{in} \quad \mathbf{R} \quad \text{as} \quad n \to \infty$$

for a suitable subsequence $\{\tau(n)\}$ with $\tau(n) \downarrow 0$ as $n \to \infty$, where P is the projection from $\mathbf{H}$ onto $\overline{D(\varphi)}$ and $[s]$ denotes the greatest integer in $s \in \mathbf{R}$.

In case $-B$ is the subdifferential of a proper l.s.c. convex functional φ on $\mathbf{H}$, i.e. $B = -\partial\varphi$, it was shown by Kato and Masuda [7] that

$$[S'(\tau)P'S(\tau)P]^{[t/\tau]}u_0 \longrightarrow u(t) \quad \text{in} \quad \mathbf{H} \quad \text{as} \quad \tau \downarrow 0 \qquad \text{for} \quad t \in [0, \infty)$$

and the convergence is uniform on $[0, T]$ for each $0 < T < \infty$, where $\{S'(\tau); 0 \leq \tau < \infty\}$ is the nonlinear contraction semigroup on $\overline{D(\psi)}$ generated by $-\partial\psi$ and P' the projection from $\mathbf{H}$ onto $\overline{D(\psi)}$. This result is a nonlinear analogue of Trotter's product formula for linear nonnegative self-adjoint operators (cf. [2, 6]) and the family $\{T(\tau); 0 \leq \tau < \infty\}$ defined by $T(t)u_0 = u(t)$, $u_0 \in \overline{D(\varphi) \cap D(\psi)}$, gives