HIROSHIMA MATH. J.
14 (1984), 227–246

Study of the behavior of logarithmic potentials by means of logarithmically thin sets

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(Received September 5, 1983)

1. Introduction and statement of results

Let R^n $(n \geqq 2)$ be the n-dimensional euclidean space. For a nonnegative (Radon) measure μ on R^n, we set

$$L\mu(x) = \int \log \frac{1}{|x-y|} d\mu(y)$$

if the integral exists at x. We note here that $L\mu$ is not identically $-\infty$ if and only if

$$\int \log(1+|y|)d\mu(y) < \infty. \tag{1}$$

Denote by $B(x, r)$ the open ball with center at x and radius r. For $E \subset B(0, 2)$, define

$$C(E) = \inf \mu(R^n),$$

where the infimum is taken over all nonnegative measures μ on R^n such that S_μ (the support of μ) $\subset B(0, 4)$ and

$$\int \log \frac{8}{|x-y|} d\mu(y) \geqq 1 \qquad \text{for every} \quad x \in E.$$

If $E \subset B(x^0, 2)$, then we set

$$C(E) = C(\{x - x^0;\ x \in E\}).$$

One notes here that this is well defined, i.e., independent of the choice of x^0.

Throughout this paper let k be a positive and nonincreasing function on the interval $(0, \infty)$ such that

$$k(r) \leqq Kk(2r) \qquad \text{for any} \quad r,\ 0 < r < 1,$$

where K is a positive constant independent of r. A set E in R^n is said to be k-logarithmically thin, or simply k-log thin, at $x^0 \in R^n$ if

$$\textstyle\sum_{j=1}^{\infty} k(2^{-j})C(E'_j) < \infty,$$

where $E'_j = \{x \in B(x^0, 2) - B(x^0, 1);\ x^0 + 2^{-j}(x - x^0) \in E\}$. If $k(r) = \log r^{-1}$ for