HIROSHIMA MATH. J.
14 (1984), 619–629

A remark on singular perturbation methods

Masayuki ITO

(Received May 15, 1984)

Abstract: We inspect singular perturbation methods, which Fife has introduced to deal with stationary problems of reaction-diffusion systems, and modify the main theorem in [1] into a more useful form.

1. Introduction

Reaction-diffusion systems arise in various fields; chemistry, ecology, population dynamics, morphogenesis, physiology and so on. One of interesting phenomena is that the systems often produce various spatial patterns of solutions. An important contribution to the mathematical illustration of such a phenomenon is made by Fife [1]. That is, under some assumptions, stationary solutions with boundary and interior transition layers are obtained constructively and rigorously by using singular perturbation techniques and matching arguments. His work itself is very attractive from a mathematical point of view. Moreover, it is recognized that his results play an important role for elucidating a complicated structure of (stationary) solution set of a type of reaction-diffusion systems (see, e.g., Mimura et al. [6], [7], Fujii et al. [3]). However, the arguments in [1] demand a hypothesis which, generally, is not expected to hold. In this paper, we intend to remove the hypothesis. In order to state our aim more precisely, we review his results.

Consider the following problem:

$$
(1.1)\qquad \begin{cases} \varepsilon^2 u'' = f(u, v), & \\ & 0 < x < 1, \\ v'' = g(u, v), & \\ u(0) = \alpha_1, \quad u(1) = \alpha_2, \quad v(0) = \beta_1, \quad v(1) = \beta_2, & \end{cases}
$$

where f, g are smooth functions, α_1, α_2, β_1, β_2 are given constants and $' = d/dx$. We make the following assumptions I $\sim$ IV.

I. The equation $f(u, v) = 0$ has two distinct solutions $u = h_0(v)$, $u = h_1(v)$, for $v \in I_0$ and $v \in I_1$, respectively, where I_i are open overlapping intervals with $\beta_{i+1} \in I_i$ ($h_0(v) < h_1(v)$ on $I_0 \cap I_1$). On I_i,

$$f_u(h_i(v), v) > 0.$$