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On generic 1-parameter families of C^∞-maps of an n-manifold into a $(2n-1)$-manifold

Dedicated to Professor Tatsuo Homma on his 60th birthday

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Introduction

Let $n \geqq 2$. It is well known that a proper C^∞-map of an n-manifold M^n into a $(2n-1)$-manifold N^{2n-1} is stable if and only if it is regular except at the isolated singularities of Whitney umbrella and its sheets intersect in general position. In this note we clarify the normal forms of stable and generic C^∞-paths connecting two stable C^∞-maps of M^n into N^{2n-1}. We put

$$\mathscr{P} = \{f\colon M \times I \to N \times I \text{ proper } C^\infty\text{-maps with } p_N \circ f = p_M;\ f_0 \text{ and } f_1 \text{ are stable}\},$$

where p_X is the projection $X \times I \to I$ for $X = M$ or N and $f(x, \lambda) = (f_\lambda(x), \lambda)$.

DEFINITION. Let $f, g \in \mathscr{P}$. We say that f is IA-equivalent to g or $f \underset{IA}{\sim} g$ if there exist diffeomorphisms k and K which satisfy the following commutative diagram (where the maps to I are the natural projections):

$$\begin{tikzcd}
M\times I \arrow[rr, "f"] \arrow[dr] \arrow[dd, "k"] & & N\times I \arrow[dl] \arrow[dd, "K"] \\
& I & \\
M\times I \arrow[rr, "g"] \arrow[ur] & & N\times I \arrow[ul]
\end{tikzcd}$$

Accordingly we call $f \underset{IA}{\sim} g$ as germs at (x, λ) if there exist such k and K as germs.

THEOREM. *Let $\mathscr{Q} = \{f \in \mathscr{P};\ f$ satisfies one of the following conditions* (i)–(iv) *as germs at any point of $M \times I\}$. Then, $\mathscr{Q}$ is an open and dense subspace of $\mathscr{P}$ with respect to the fine C^∞-topology of $\mathscr{P}$ which is defined by the identification with a subspace of $C^\infty(M^n \times I, N^{2n-1})$.*

(i) $f \underset{IA}{\sim} (x_1, \dots, x_n, 0, \dots, 0, \lambda)$,
(ii) $f \underset{IA}{\sim} (x_1^2, x_2, \dots, x_n, x_1x_2, \dots, x_1x_n, \lambda)$ (*Whitney umbrella* $\times\, \mathbf{R}$),
(iii) $f \underset{IA}{\sim} (x_1^2, x_2, \dots, x_n, x_1(\pm\lambda - x_1^2 - x_2^2), x_1x_3, \dots, x_1x_n, \lambda)$ *and*
(iv) $f \underset{IA}{\sim} (x_1^2, x_2, \dots, x_n, x_1(\pm\lambda + x_1^2 - x_2^2), x_1x_3, \dots, x_1x_n, \lambda)$.