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Boundedness of singular integral operators of Calderón type (IV)

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1. Introduction

We denote by L^p $(1 \leqq p \leqq \infty)$ the L^p-space on the real line $\boldsymbol{R}$ with norm $\|\cdot\|_p$ with respect to the 1-dimensional Lebesgue measure $|\cdot|$. We denote by S^∞ the totality of rapidly decreasing functions on $\boldsymbol{R}$. We say that a locally integrable function $f(x)$ is of bounded mean oscillation if $\|f\|_{BMO} = \sup (1/|I|) \int_I |f(x) - m_I f|\, dx < \infty$, where $m_I f = (1/|I|) \int_I f(x)\, dx$ and the supremum is taken over all finite intervals I. The space BMO of functions of bounded mean oscillation, modulo constants, is a Banach space with norm $\|\cdot\|_{BMO}$. For $0 < \delta \leqq 1$ and a complex-valued kernel $K(x, y)$ $(x, y \in \boldsymbol{R})$, we define $\omega_\delta(K)$ by the infimum over all A's with the following three inequalities:

$$|K(x, y)| \leqq A/|x-y| \quad (x \neq y)$$

$$|K(x, y) - K(x', y)| \leqq A|x-x'|^\delta/|x-y|^{1+\delta} \quad (|x-x'| \leqq |x-y|/2,\ x \neq y)$$

$$|K(x, y) - K(x, y')| \leqq A|y-y'|^\delta/|x-y|^{1+\delta} \quad (|y-y'| \leqq |x-y|/2,\ x \neq y).$$

(If such an A does not exist, we put $\omega_\delta(K) = \infty$.) We say that $K(x, y)$ is a Calderón-Zygmund kernel (CZ-kernel), if $\omega_\delta(K) < \infty$ for some $0 < \delta \leqq 1$,

$$Kf(x) = \int_{-\infty}^{\infty} K(x, y) f(y)\, dy = \lim_{\varepsilon \to 0} \int_{|x-y|>\varepsilon} K(x, y) f(y)\, dy$$

exists almost everywhere (a.e.) for any $f \in L^2$ and $\|K\| = \sup\{\|Kf\|_2/\|f\|_2; f \in L^2\} < \infty$. For a CZ-kernel $K(x, y)$, a complex-valued function $h(x)$ and a real-valued function $\phi(x)$, we put

$$K[h, \phi](x, y) = K(x, y) h\left\{\frac{\phi(x) - \phi(y)}{x - y}\right\}.$$

Calderón [1] showed that $K[h, \phi]$ is a CZ-kernel if $K(x, y) = 1/(x-y)$, $\phi' \in L^\infty$ and $h(x)$ is extended as an entire function, where "$\phi' \in L^\infty$" implies that $\phi(x)$ is differentiable a.e. and its derivative is essentially bounded. Coifman-David-Meyer [4] showed that Calderón's theorem is valid with the above condition on $h(x)$ replaced by "$h \in S^\infty$". The author [7] showed that their theorem