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Remarks on the separation of the *Aa*-adic topology and permutations of *M*-sequences

Michinori SAKAGUCHI

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1. Introduction

Let M be a non-zero, finite module over a noetherian ring A. It is well known that if A is a local ring with the maximal ideal $\mathfrak{m}$, then every permutation of an M-sequence is an M-sequence. It seems to the author that this property arises from the fact that the $\mathfrak{m}$-adic topology on M is a Hausdorff space. In this paper we study modules M which satisfy the condition that the Aa-adic topology on M is separated for every M-regular element a. As a tool in this investigation we consider the subset $\mathscr{K}(M)$ of A which consists of those elements a with separated Aa-adic topology.

In section 2 we study some inclusion relations among the set $\mathscr{K}(M)$, the set of all zero-divisors of M and the set of all M-regular elements. In section 3 we establish a method of constructing modules M such that the sets $\mathscr{K}(M)$ are as large as possible. In section 4 we give some conditions equivalent to the assertion that the sequence $\{b, a\}$ is an M-sequence for every M-sequence $\{a, b\}$.

All rings are assumed to be noetherian, commutative, with unity, and all modules are assumed to be of finite type, unitary.

Let A be a ring and M an A-module. We write $\mathscr{Z}(M)$ for the set of zero-divisors on M. Let a be an element of A and let f_a be the homomorphism $M \xrightarrow{a} M$, where $f_a(m)=am$ for $m \in M$. Then $a \in \mathscr{Z}(M)$ if and only if f_a is not injective. We denote by $\mathscr{R}(M)$ the set of M-regular elements. Note that $a \in \mathscr{R}(M)$ if and only if f_a is injective but not surjective. We let $\mathscr{U}(M)$ denote the set of all elements a in A such that f_a are isomorphims. If M is a non-zero module, it is clear that A is a disjoint union of the subsets $\mathscr{Z}(M)$, $\mathscr{R}(M)$ and $\mathscr{U}(M)$. Further we use freely the terminologies in [2].

2. The set $\mathscr{K}(M)$

DEFINITION. Let A be a ring, M an A-module. Then the set $\mathscr{K}(M)$ is defined to be the set of those elements a of A such that $\bigcap_{n=1}^{\infty} a^n M = 0$.

It follows easily from our definition that $\mathscr{K}(M) \subset \mathscr{Z}(M) \cup \mathscr{R}(M)$ for a non-zero A-module M. In general $\mathscr{K}(M)$ is not an ideal. Applying Krull's intersection theorem, we have a basic proposition about $\mathscr{K}(M)$.