HIROSHIMA MATH. J.
15 (1985), 69–74

Asymptotic nonoscillation under large amplitudes of oscillating coefficients in second order functional equations

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(Received June 15, 1982. Revised June 28, 1984)

1. Introduction

Our main purpose in this work is to study the nonoscillation property of the solutions of the functional equation

$$y''(t) + a(t)y^{\delta}(g(t)) = f(t) \tag{1}$$

when $a(t)$ oscillates with sufficiently large amplitudes. In the sequel we shall assume that

(i) $a(t)$, $g(t)$, $f(t)$: $R \to R$ and continuous;

(ii) $g(t) \to \infty$ as $t \to \infty$, $g'(t) > 0$ (thus $g(t)$ may be advanced or retarded);

(iii) $\delta > 0$ and the ratio of odd integers. The equation (1) is therefore, allowed to be superlinear, linear or sublinear.

In view of our Theorem 3.1 in [5], we shall assume that solutions of equation (1) under consideration are those which exist continuously on some half line $[t_0, \infty)$. The term "solution", henceforth, applies only to such an entity.

Since the pioneering work of Hammett [1] the asymptotic nature of oscillatory and nonoscillatory solutions of equation (1) has been the subject of numerous studies. A recently published Russian book by Shevelo gives a fairly exhaustive list of references for the interested reader. In regard to obtaining results about the asymptotic nature of the nonoscillatory solutions, the coefficient $a(t)$ has been assumed to be of one sign by majority of authors. When $a(t) > 0$ for sufficiently large t, then a decent account of the nonoscillatory solutions can be found in Kusano and Onose [3–4], Kitamura Kusano and Naito [2], Singh [6] and Kusano and Singh [7]. However when $a(t)$ is oscillatory, nothing seems to be known about the asymptotic nature of the nonoscillatory solutions of equation (1).

In this work, we present an elementary but new technique to study oscillation phenomenon in general. In particular, we not only assume that $a(t)$ be oscillatory, but also utilize the amplitude of oscillation to characterize the nonoscillatory solutions of (1). In what follows we call a solution of (1) oscillatory if it has arbitrarily large zeros in $[t_0, \infty)$; otherwise we call it nonoscillatory.