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Study of three-dimensional algebras with straightening laws which are Gorenstein domains II

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Introduction

In the previous paper [6] we determined all the partially ordered sets (poset for short) on which there exist three dimensional homogeneous Gorenstein ASL (algebra with straightening laws) domains over a field. Now, in this second part, we shall analyze normality and rationality of these algebras.

Our main purpose in this paper is to prove the following

THEOREM. *Let k be an algebraically closed field of arbitrary characteristic.*

(i) *The non-normal three dimensional homogeneous Gorenstein ASL domain over k is, up to isomorphism as ASL, either* [6, Example g)] *or* Example b) *in* §3.

(ii) *Every three dimensional homogeneous Gorenstein ASL domain over k is rational, that is, the quotient field of this algebra is a purely transcendental extension of the base field k.*

The basic methods in our proof are the calculations of singularities and the theory of "branches" (see §4). The former is useful to find out the non-normal ASL domains, while the latter plays an essential role for the proof of rationality.

Moreover, the calculations of singularities enable us to classify all the homogeneous ASL domains on the poset C_6 (§1). This classification is accomplished by means of some expressions of these algebras as subalgebras of the Veronese subring $k[x, y, z]^{(3)}$ (see (2.4) in §2). We will continue our classification in our further work.

Apart from the above results, this paper contains several lemmas, especially Lemma 10 in §4, which give criteria for a quasi-ASL (§1) to be an ASL. Using these lemmas it is easy to see that all the examples appeared in [6] are ASL.

§1. Notation and preliminaries

We shall refer to [6] for the basic definitions and terminologies on commutative algebras and combinatorics and, unless otherwise stated, keep the notation in [6]. We here summarize additional notation and results which are not contained in [6].