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Quasi-artinian groups

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Introduction

Aldosray [1] introduced the concept of quasi-artinian Lie algebras generalizing those of soluble Lie algebras and artinian Lie algebras, that is, Lie algebras satisfying the minimal condition for ideals, and left an open question asking whether a semisimple quasi-artinian Lie algebra is always artinian. On the other hand, he introduced the concept of quasi-artinian groups in an analogous way and noted that the corresponding results mentioned in [1] hold for groups. Subsequently Kubo and Honda [2] provided a negative answer to the question above, and moreover gave a condition under which quasi-artinian Lie algebras are soluble (resp. artinian).

In this paper, following the paper [2] we construct a semisimple quasi-artinian group which is neither soluble nor artinian and give a condition under which quasi-artinian groups are soluble (resp. artinian).

We shall prove in Section 2 that the class of quasi-artinian groups is countably recognizable (Proposition 2.2) and that a subgroup with finite index in a quasi-artinian group is quasi-artinian under some conditions (Proposition 2.3). In Section 3 we shall prove that every residually (ω)-central quasi-artinian group is soluble (Theorem 3.3) and that every residually commutable quasi-artinian group is hyperabelian (Theorem 3.7). The main result of Section 4 is that a quasi-artinian group G is artinian if and only if for each normal subgroup N of G G/N satisfies the minimal condition on abelian normal subgroups (Theorem 4.2). In Section 5 we shall give examples showing that the class of quasi-artinian groups is not E-closed (i.e. $\boldsymbol{P}$-closed) and is not s_n-closed.

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1.

Let G be a group. As usual, $x^y=y^{-1}xy$ and $[x, y]=x^{-1}y^{-1}xy$, $[x, y, z]=[[x, y], z]$ for $x, y, z\in G$. We write inductively

$$\mathrm{D}^1(x_1, x_2) = [x_1, x_2],$$

$$\mathrm{D}^{n+1}(x_1,\dots, x_{2^{n+1}}) = [\mathrm{D}^n(x_1,\dots, x_{2^n}), \mathrm{D}^n(x_{2^n+1},\dots, x_{2^{n+1}})] \quad (n\geq 1),$$