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Uniform simplification in a full neighborhood of a turning point

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§ 1. Introduction

In this paper we shall consider the system of linear ordinary differential equations with a parameter

$$\varepsilon \frac{dX}{dt} = A(t, \varepsilon)X, \tag{1.1}$$

where ε is a complex parameter, t is a complex variable and X is an unknown vector function of t and ε. Let t_0, ε_0 and θ_0 be positive constants. We shall introduce the following assumptions.

(i) $A(t, \varepsilon)$ is an n by n matrix function of t and ε which is holomorphic in the domain:

$$D(t_0, \varepsilon_0, \theta_0) = \{(t, \varepsilon) \mid |t| \leqq t_0,\ 0 < |\varepsilon| \leqq \varepsilon_0,\ |\arg \varepsilon| \leqq \varepsilon_0\}\ ;$$

(ii) $A(t, \varepsilon)$ admits an asymptotic expansion:

$$A(t, \varepsilon) \simeq \sum_{i=0}^{\infty} A_i(t)\varepsilon^i$$

uniformly for $|t| < t_0$, as ε tends to zero in the sector

$$0 < |\varepsilon| \leqq \varepsilon_0,\ |\arg \varepsilon| \leqq \theta_0, \tag{1.2}$$

where each $A_i(t)$ is holomorphic in the closed disk $|t| \leqq t_0$;

(iii) the function $A_0(t)$ has the form

$$A_0(t) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \\ \vdots & \vdots & \vdots & \ddots & 1 \\ t^q & 0 & 0 & \cdots & 0 \end{pmatrix},$$

where q is a positive integer.

Assumption (iii) means that $t=0$ is a turning point of order q of the differential equation (1.1) and there is no other turning point in the closed disk $|t| \leqq t_0$. In order to investigate the asymptotic behavior of solutions of the system (1.1) in a