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On the Adams-Novikov spectral sequence and products of β-elements

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§1. Introduction

Let p be a given prime $\geqq 5$ and BP the Brown-Peterson spectrum at p; and consider the Hopf algebroid (cf. [2], [12])

$$(A, \Gamma) = (BP_*, BP_*BP) = (Z_{(p)}[v_1, v_2, \cdots], BP_*[t_1, t_2, \cdots]),$$

and the Γ-comodules A and $A/(p)$. Then, for the sphere spectrum S localized at p and the Moore spectrum M mod p, we have the Adams-Novikov spectral sequence (cf. [3], [12]):

$$(1.1) \qquad E_2 = \mathrm{Ext}_\Gamma^*(A, A) \quad (\text{resp. } \mathrm{Ext}_\Gamma^*(A, A/(p))) \Longrightarrow \pi_*S \quad (\text{resp. } \pi_*M).$$

This is investigated by several authors to study the structure of the stable homotopy ring π_*S of spheres ([3], [6], [7]).

Now, for the Γ-comodules N_1^j and $M_1^j = v_{1+j}^{-1}N_1^j$ such that $N_1^0 = A/(p)$ and N_1^{j+1} is the cokernel of the localization map $N_1^j \to M_1^j$, we have the chromatic spectral sequence (cf. [3]):

$$(1.2) \qquad E_2 = \mathrm{Ext}_\Gamma^*(A, M_1^*) \Longrightarrow \mathrm{Ext}_\Gamma^*(A, A/(p)).$$

In this paper, we are concerned with $\mathrm{Ext}_\Gamma^*(A, M_1^1)$ for $* \geqq 2$ by continuing the studies in [3] and [11] for $* = 0, 1$ to obtain the following

THEOREM A. *The $F_p[v_1]$-module $\mathrm{Ext}_\Gamma^*(A, M_1^1)$ is given by Theorem 4.4.*

Here, we note the following: Consider the spectrum N which is the cofiber of the localization map $M \to \alpha^{-1}M$ for the Adams map $\alpha \in [M, M]_*$. Then, by Ravenel's localization functor L_2 (see [10]), we have the spectrum L_2N with $BP \wedge L_2N = N \wedge v_2^{-1}BP$ and the Adams-Novikov spectral sequence:

$$(1.3) \qquad E_2 = \mathrm{Ext}_\Gamma^*(A, M_1^1) \Longrightarrow \pi_*(L_2N).$$

Thus, Theorem A implies immediately the following

COROLLARY. *The spectral sequence (1.3) collapses, and $\pi_*(L_2N)$ is an $F_p[\alpha]$-module isomorphic to $\mathrm{Ext}_\Gamma^*(A, M_1^1)$ in Theorem 4.4 by sending α to v_1.*