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Asymptotic behavior of solutions of a class of second order nonlinear differential equations

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1. Introduction

This paper is concerned with the asymptotic behavior of solutions of the second order nonlinear differential equation

$$(p(t)y')' + f(t, y, y') = 0, \tag{1}$$

for which the following conditions are assumed to hold:

(A_1) p: $[0, \infty) \to (0, \infty)$ is continuous, and

$$P(t) = \int_0^t \frac{ds}{p(s)} \to \infty \quad \text{as} \quad t \to \infty; \tag{2}$$

(A_2) f: $[0, \infty) \times \boldsymbol{R} \times \boldsymbol{R} \to (0, \infty)$ is continuous and nondecreasing in each of the last two variables.

A prototype of equation (1) satisfying (A_1) and (A_2) is

$$y'' + \varphi(t)e^y = 0, \tag{3}$$

or more generally

$$y'' + \varphi(t)\exp(|y|^{\gamma-1}y + |y'|^{\delta-1}y') = 0, \tag{4}$$

where φ: $[0, \infty) \to (0, \infty)$ is continuous and γ and δ are positive constants. It seems to us that no systematic study of the qualitative behavior of solutions has so far been attempted even for the simple equation (3) or (4), and this observation motivated the present work.

We begin by noticing that all solutions of (1) can be continued to infinity. In fact, let $y(t)$ be a solution of (1) with given initial values at $t = a$ ($a \geqq 0$) and let $[a, T)$ be its right maximal interval of existence. Suppose that $T < \infty$. From (1), $(p(t)y'(t))' = -f(t, y(t), y'(t)) < 0$ on $[a, T)$, so that $p(t)y'(t)$ is decreasing and tends to $-\infty$ as $t \to T^-$. Hence there exist constants $t_0 \in (a, T)$, k and l such that $y(t) \leqq k$ and $y'(t) \leqq l$ on $[t_0, T)$. Integrating (1) on $[t_0, t]$, we have

$$p(t_0)y'(t_0) - p(t)y'(t) = \int_{t_0}^t f(s, y(s), y'(s))ds \leqq \int_{t_0}^t f(s, k, l)ds,$$

which, in the limit as $t \to T^-$, gives