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# Non-triviality of some compositions of $\beta$-elements in the stable homotopy of the Moore spaces

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## §1. Introduction

Let $S$ be the sphere spectrum and $M$ the Moore spectrum modulo a prime $p \geqq 5$ given by the cofiber sequence $S \xrightarrow{p} S \xrightarrow{i} M \xrightarrow{\pi} \Sigma S$; and consider the stable homotopy rings $\pi_* S$ and $[M, M]_*$. Then, for $s \geqq 1$ and $t \geqq 2$, the $\beta$-elements

(1.1) $\quad \beta_{(s)}, \quad \beta_{(tp/p)}$ in $[M, M]_*$ and $\beta_s = \pi\beta_{(s)}i$,

$$\beta_{tp/p} = \pi\beta_{(tp/p)}i, \quad \beta_{tp^2/p,2} \text{ in } \pi_* S$$

are given by Smith [13] (see also [14], [16]) and Oka [7], [8].

Consider the Brown-Peterson spectrum $BP$ at $p$, the Hopf algebroid $(A, \Gamma) = (BP_*, BP_*BP) = (Z_{(p)}[v_1, v_2, \cdots], BP_*[t_1, t_2, \cdots])$ and the Adams-Novikov spectral sequence:

$$E_2 = H^*A' = \mathrm{Ext}_\Gamma^*(A, A') \Longrightarrow \pi_* M \text{ (resp. } \pi_* S) \qquad \text{for} \quad A' = A/(p) \text{ (resp. } A).$$

Then, Miller-Ravenel-Wilson [4] proved the following:

(1.2) There are the $\beta$-elements

$$\beta'_s \quad \text{in} \quad H^1 A/(p) \text{ (resp. } \beta_s, \beta_{tp/p}, \beta_{tp^2/p,2} \text{ in } H^2 A) \text{ (see (2.4.6))}$$

which converge to $\beta_{(s)}i$ in $\pi_* M$ (resp. the ones in $\pi_* S$ with the same notation).

The main purpose of this paper is to prove the following

THEOREM A. *In the $E_2$-term $H^3 A/(p)$, $\beta'_s\beta_{tp^2/p,2} = \beta'_{s+tp(p-1)}\beta_{tp/p}$ holds, and $\beta'_s\beta_{tp/p} = 0$ if and only if $p|st$.*

COROLLARY B. *In $[M, M]_*$, $\beta_{(s)}(\beta_{tp^2/p,2} \wedge 1_M)$, $\beta_{(s)}(\beta_{tp/p} \wedge 1_M)$ and $\beta_{(s)}\delta\beta_{(tp/p)}$ are all non-trivial if $p \nmid st$. Here $\delta = i\pi$ is the generator of $[M, M]_{-1}$.*

Corollary B is a consequence of Theorem A and is proved in Corollary 4.2. The equality and the triviality in Theorem A are in Theorem 2.7 which is valid for $p \geqq 3$ and can be proved easily by [4] and [9], and the non-triviality is in Theorem 4.1. We note that Theorems 2.7, 4.1 and Corollary 4.2 contain the (non-) triviality of some other compositions.