HIROSHIMA MATH. J.
16 (1986), 115–120

A characterization of Prüfer v-multiplication domains in terms of polynomial grade

Hirohumi UDA

(Received December 25, 1984)
(Revised April 18, 1985)

Prüfer v-multiplication domains, abbreviated to PVMD's, have among their special cases a variety of notions, including Prüfer domains, Krull domains, GCD domains, etc. Many interesting characterizations of PVMD's are given by several authors (see [2], [3], [5], [7], [8], [11]). The main purpose of this paper is to give a characterization of PVMD's in terms of polynomial grade (cf. Theorem 2 and Remark 3). This characterization makes the situation of PVMD's in the class of P-domains clearer.

Moreover, we shall examine some properties of PVMD's by making use of Theorem 2 and Remark 3. First, we shall give some characterizations of PVMD's in the class of intergrally closed domains (cf. Theorem 5 and Proposition 7). In particular, Theorem 5 is a generalization of Theorem 3.4 of [5]. Next, we shall give a necessary and sufficient condition for an FC domain to be integrally closed (cf. Proposition 11). Finally, in case A is a PVMD, we shall give a characterization of G_2-stableness of $A \subset B$, where B is an overring of A (cf. Proposition 12).

To give our results, we include the following notions and notations.

Throughout this paper, A and K denote an integral domain and its quotient field respectively. Moreover, we denote by X an indeterminate. For a fractional ideal I of A, we put $I_v = A :_K (A :_K I)$. We say that I is a *v-ideal* if $I = I_v$, and a v-ideal I is *of finite type* if there is a finitely generated fractional ideal J of A such that $I = J_v$. An integral domain A is called a *Prüfer v-multiplication domain (PVMD)*, if the set of all v-ideals of A of finite type forms a group under the v-multiplication $I \cdot J = (IJ)_v$, [3]. Let I be an ideal of A. We denote by $\mathrm{gr}(I)$ and $\mathrm{Gr}(I)$ the classical grade of I and the polynomial grade of I respectively, [6]. The following subsets of $\mathrm{Spec}(A)$ are needed for this paper.

$$\mathfrak{P}(A) = \{P \in \mathrm{Spec}(A) \mid P \text{ is minimal over } a\colon b \text{ for some } a, b \in A\}.$$

$$\mathfrak{G}(A) = \{P \in \mathrm{Spec}(A) \mid \mathrm{Gr}(P) \leqq 1\}.$$

If A_P is a valuation ring for each $P \in \mathfrak{P}(A)$, A is called a *P-domain*, [5]. It is known that a PVMD is a P-domain, ([5], Corollary 1.4). Since $A = \cap \{A_P \mid P \in \mathfrak{P}(A)\}$ by Theorem E of [9] and $\mathfrak{P}(A) \subset \mathfrak{G}(A)$, we have $A = \cap \{A_P \mid P \in \mathfrak{G}(A)\}$.

Let I be an ideal of $A[X]$. We denote by $c(I)$ the ideal of A generated by